Name:
Pledge:

Math 411 Final

Due Wednesday May 2 at 12:30pm.
Slip exam under door of my office (Roop 339).
You may use your book, your notes, and me for help.
No other sources.

(1) (Part I - 15 points)

(a) If E is measurable, show that $E + x$ is measurable.

(b) Let $f : \mathbb{R} \to \mathbb{R}$. If f is measurable, show that $g(x) = f(x + t)$ is measurable, for any t.

(c) If $f \in \mathcal{L}_1$, show $g \in \mathcal{L}_1$ and $\int f = \int g$.

(2) (Part II - 15 points)

(a) Let $f \in \mathcal{L}_1(\mathbb{R})$. Carefully prove that

$$\lim_{n\to\infty} \int_{-\infty}^{\infty} f(x) \cos(nx) dx = 0.$$

(b) Let $E \subseteq \mathbb{R}$ be closed, let $f : E \to \mathbb{R}$ be continuous. Construct a continuous function $g : \mathbb{R} \to \mathbb{R}$ that agrees with f on E.

(c) Let (f_n) be a sequence of measurable functions converging pointwise a.e. to a real-valued function f on a measurable set D of *arbitrary* measure. Show there exist measurable sets $E_1 \subseteq E_2 \subseteq \cdots \subseteq D$ so that $(f_n) \to f$ uniformly on each E_k and $m\left(\left(\cup_{k=1}^{\infty} E_k\right)^c\right) = 0$.

(3) (Bonus!) 1 point each.

(a) Tell me something you learned from one of the presentations. (Not your own.)

(b) Let $1 < p < q < \infty$. Show that $\mathcal{L}_p \not\subset \mathcal{L}_q$.

(c) Let $1 < p < q < \infty$. Show that $\mathcal{L}_q \not\subset \mathcal{L}_p$.