Name:
Pledge:

Math 411 Midterm

Due Friday March 30 at start of class.
You may use your book, your notes, and/or me for help.
No other sources.

(1) (Part I - 15 points)
   (a) Show that the simple integrable functions are dense in L_1 by applying the Dominated Convergence Theorem to our basic construction (Lebesgue Ladder).

   (b) Let $f, f_n \in L_1[a, b]$. Show that $f_n \to f$ uniformly on $[a, b]$ implies that $f_n \to f$ in $L_1[a, b]$. Is the result true if $[a, b]$ is replaced by $\mathbb{R}$?

   (c) Suppose the sequence of measurable functions $\{f_n\}$ is cauchy in measure, and that some subsequence $\{f_{n_k}\}$ converges in measure to a measurable function f. Show that $f_n \to f$ in measure.

(2) (Part II - 15 points)
   Prove that L_1 is complete by following the steps below.
   (a) Show that any cauchy sequence $\{f_n\}$ in L_1 is cauchy in measure. (Use Chebyshev.)

   (b) Given a sequence $\{f_n\}$ cauchy in measure, apply the theorem of Riesz to produce a function f to which some subsequence $\{f_{n_k}\}$ converges pointwise almost everywhere. Show that $f_n \to f$ in L_1. (Use Fatou to estimate $\|f_{n_k} - f\|$.)

   (c) Put all of this together for a complete proof, remembering to check any and all details.

(3) (Bonus!) Compute the limits in exercise 44, page 332.
   (1/2 point for each.)