Rota's Basis Conjecture

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This conjecture is stated for any finite dimensional vector space over any field.

One way of looking at the problem is to view the original bases, $\{a_1, a_2, \ldots, a_n\}, \{b_1, b_2, \ldots, b_n\}, \ldots, \{k_1, k_2, \ldots, k_n\}$ as the rows of an array:

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$$\begin{array}{cccc} a_1 & a_2 & \cdots & a_n \\ b_1 & b_2 & \cdots & b_n \\ \vdots & \vdots & & \vdots \\ k_1 & k_2 & \cdots & k_n \end{array}$$

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Rota's Basis Conjecture asserts that there is a way to permute the entries of each row of this array so that each of the resulting columns forms a basis.

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Rota's Basis Conjecture states that one can independently permute each row, so that all the columns form a basis. That is, each new column basis will contain exactly one vector of each color, forming a "rainbow basis."

The Conjecture

Example: Let n=3. We can use vectors from $\mathbb{R}^3$ to form this matrix :

$$\begin{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} & \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} & \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \end{bmatrix}$$

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After permuting the rows, we can now see that all columns do indeed form basis' and we have verified Rota's Basis Conjecture for this example.

Alon Tarsi Conjecture ('92): For Latin Squares of **even** size n the number of even Latin Squares of size n and the number of odd Latin Squares of size n are different.

Progress Made so Far

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We look at this for every change in number. Then we take $(\text{-}1)^{\#\text{ of inversions in each row}}$.
We then repeat this for the columns and get either 1 or (-1) for each row and column.
Finally we multiply all the 1's and (-1)'s together.
If we get 1 = even L.S and (-1) = odd L.S.

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Has also, been verified for n $=$ p $+$ 1 for all odd prime p (Drisko, 1995), and n $=$ p - 1 for all odd prime p (Glynn, 2010).
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Wendy Chan ('95): Solved for $n = 3$ bases in a rank 3 matroid. Used the Basis Exchange Theorem and solved the conjecture using 3 cases.

Famous Rota Quotes

"God created infinity, and man, unable to understand infinity, had to invent finite sets."

"A mathematician's work is mostly a tangle of guesswork, analogy, wishful thinking and frustration, and proof, far from being the core of discovery, is more often than not a way of making sure that our minds are not playing tricks."

"We often hear that mathematics consists mainly of 'proving theorems.' Is a writer's job mainly that of 'writing sentences'?"

Computational Proof of Rota's Basis Conjecture for Matroids

Michael S. Cheung

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Matroids

A **Matroid** has many equivalent definitions; the one most convenient for our purposes is the basis formulation:

Definition (Matroid)

A **Matroid** is an ordered pair $M = (S, \mathcal{B})$, where S is a set and $\mathcal{B}$ is a collection of subsets of S (called the **bases** of M), that satisfies the following properties:

M1: $\mathcal{B}$ is nonempty.

M2 (Basis Exchange):

$\forall B_1, B_2 \in \mathcal{B}, A_1 \subset B_1, \exists A_2 \subset B_2 \mid (B_1 - A_1) \cup A_2, (B_2 - A_2) \cup A_1 \in \mathcal{B}$

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Matroid Example

$$
\begin{array}{cc}
\text{members:} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\
 & \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}
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$S = \{1, 2, 3, 4\}$

$\mathcal{B} = \{\{1, 2, 3\}, \{1, 3, 4\}, \{2, 3, 4\}\}$

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M2 (Basis Exchange)

Allows us to swap elements between two bases to form new bases.

We choose two bases B_1, B_2 and a subset of the first basis A_1, but no guarantees are made about A_2

We refer to a choice (B_1, B_2, A_1) as a 3-tuple and the potential A_2's as candidates.

We use the following notation: $B_1(A_1)/B_2(A_2)$

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For $n = 3$, we are given bases
$B_1 = \{a_1, a_2, a_3\}, B_2 = \{b_1, b_2, b_3\}, B_3 = \{c_1, c_2, c_3\}$

For the 3-tuple $(B_1, B_2, \{a_1, a_2\})$, possible candidates for A_2 are $\{b_1, b_2\}, \{b_1, b_3\}, \{b_2, b_3\}$. Hence, there are three cases - one for each candidate.

Case 1: $B_1(\{a_1, a_2\})/B_2(\{b_1, b_2\})$ provides two new bases:
$B_4 = \{a_3, b_1, b_2\}, B_5 = \{a_1, a_2, b_3\}$

Now we examine the 3-tuple $(B_5, B_3, \{b_3\})$, which has candidates $\{c_1\}, \{c_2\}, \{c_3\}$. Hence, we have another three cases (Case 1-1, 1-2, 1-3).

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Conjecture (Rota's Basis Conjecture for Matroids)

Let M be a matroid of rank n and let $\mathcal{B}^* = B_1, B_2, ..., B_n$ be bases in M. Then there exists n pairwise disjoint transversals of $\mathcal{B}^*$ that are bases.

- If we think of each of the n given bases as having a color, we can call these pairwise disjoint transversals "rainbow" bases.

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Wendy Chan's Approach

Wendy Chan proved the generalized conjecture for $n = 3$.

- Case 1: Two members of the matroid are dependent
- Case 2: The join of two elements of one basis is equal to that of another
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My Approach

- Forget about special cases and lemmas; use only basis exchange
- Start with the given n bases, "complete" the matroid step by step by choosing 3-tuples and using basis exchange
- Once we have a full set of disjoint rainbow bases, we have proven the conjecture
- If the matroid is "completed" without proving the conjecture, we have a counter-example

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Issues

- $\binom{n}{k}$ A_2's (where k is the order of A_1), each corresponding to a new case.
- Large number of matroids (over $10,000,000$ for $n = 3$; too large to compute for $n = 4$)
- $2^n - 2$ non-trivial A_1's, up to $\binom{n^2}{n}$ bases, meaning up to $\binom{n^2}{n}^2 \cdot (2^n - 2)$ 3-tuples ($42,336$ for $n = 3$; $46,373,600$ for $n = 4$)

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Eliminating 3-Tuples

Some 3-tuples (B_1, B_2, A_1) do not need to be considered.

- If the basis exchange property is already satisfied (so no new bases can be inferred).

Example

$B_1 = \{a_1, a_2\}, B_2 = \{a_1, b_2\}, A_1 = \{a_1\} \implies A_2 = \{a_1\}$ The basis exchange property simply swaps the two a_1's and returns the same two bases.

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We compare two 3-Tuples with the following criteria, from most to least important:

- Prefer less cases
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Identical Cases

We formalize the concept of two cases being similar (so that only one must be considered):

Two cases result in two sets of bases $\mathcal{B}_1, \mathcal{B}_2$ where one contains the other after permutation of the elements of the matroid.

Example

Consider $B_1 = \{a_1, a_2\}, B_2 = \{b_1, b_2\}, A_1 = \{a_1\}$; we have candidates $\{b_1\}, \{b_2\}$ that produce these two sets of bases:

$\{a_1, a_2\}$	$\{a_1, a_2\}$
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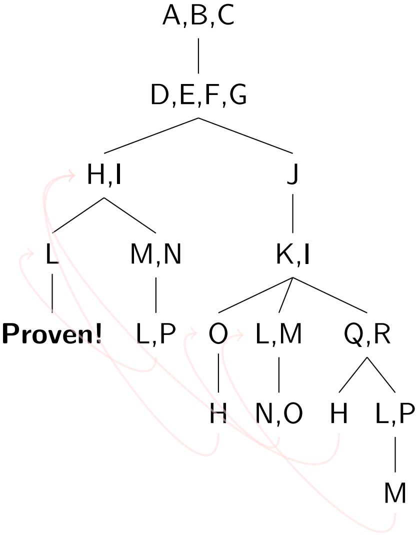

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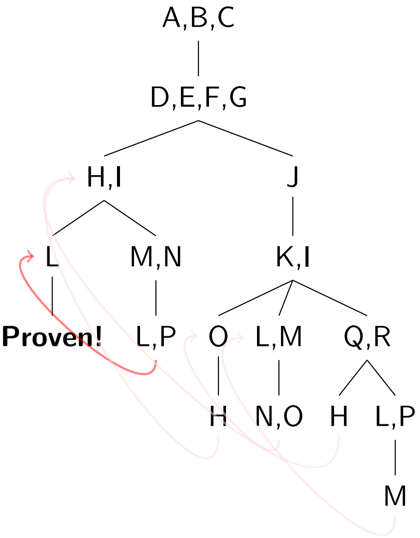

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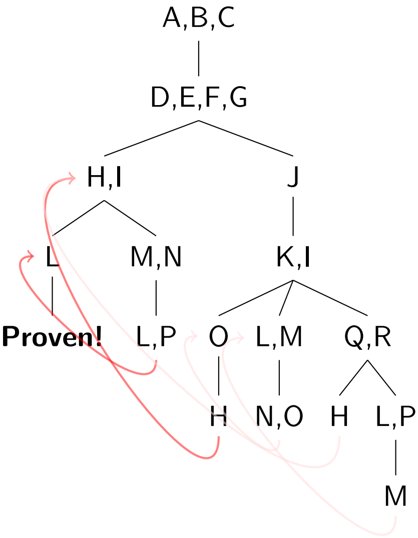

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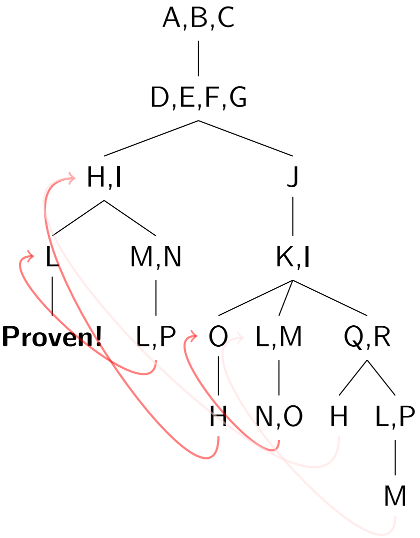

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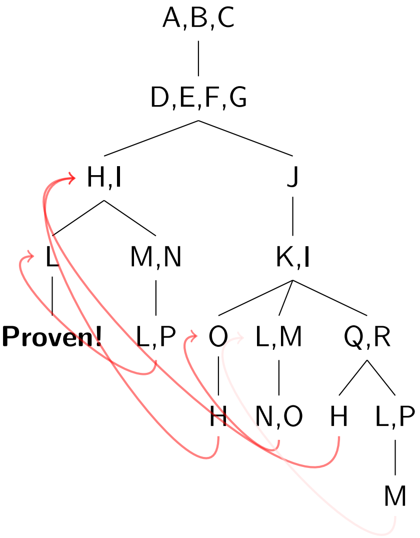

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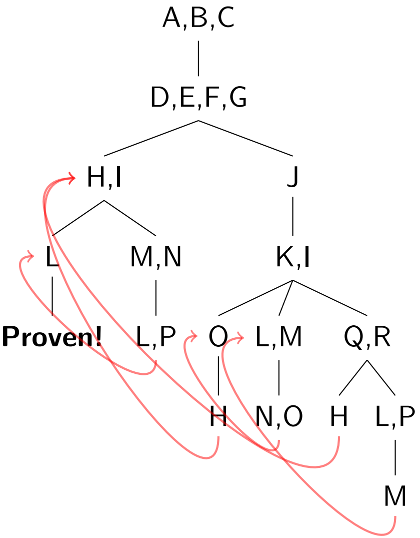

Proving the Conjecture

- Eliminate candidates that produce a set of bases proving the conjecture.
- If all candidates eliminated, then the conjecture is proven for this case

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- Program can be run for any n
- Proves $n = 3$ with only one case (as opposed to Wendy Chan's 3 cases)
- Halfway done proving $n = 4$ (around 20,000 cases considered)
- $n = 3$ the only case that has been proven for the vector space or even the generalized matroid conjecture

$n = 3$ proof

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