Gluing mASD connections on cylindrical end 4-manifolds

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(joint w/ Ian Hambleton)

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Set-up

- X = a connected, oriented 4-manifold with cylindrical ends:

$$X = X_0 \cup_N (N \times [0, \infty))$$

- E = a G-bundle over X

- A connection A is **ASD** if $F_A^+ = 0$.
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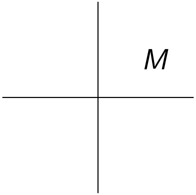

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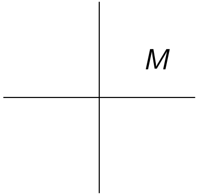

This can be understood by the presence of asymptotic limits (flat connections) that are degenerate.

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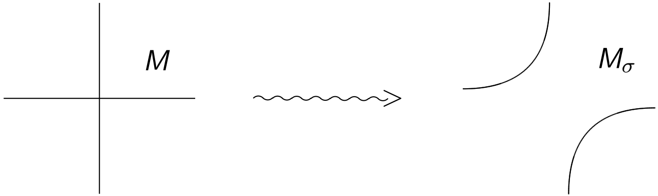

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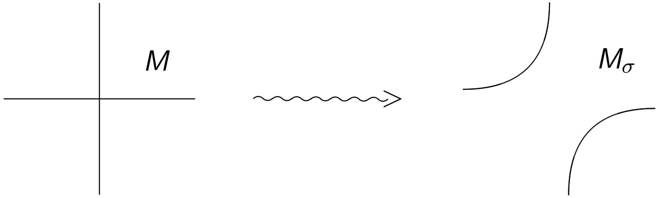

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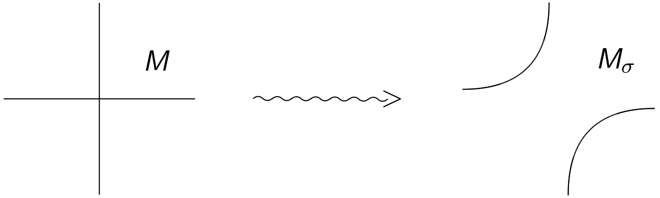

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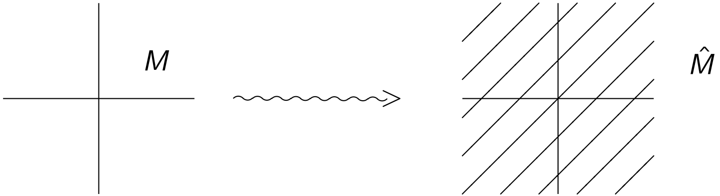

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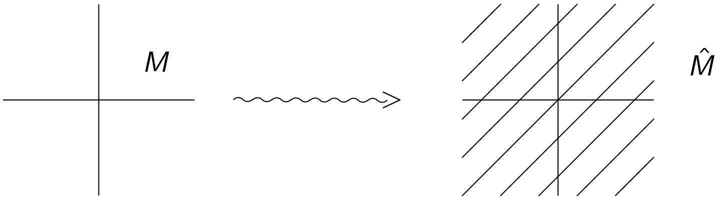

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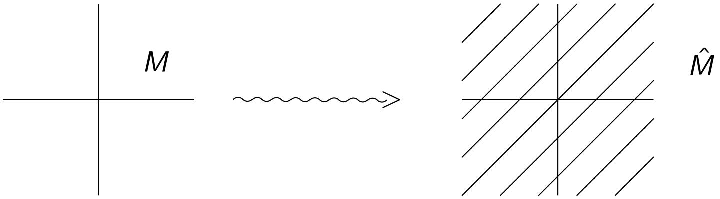

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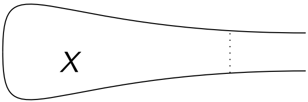

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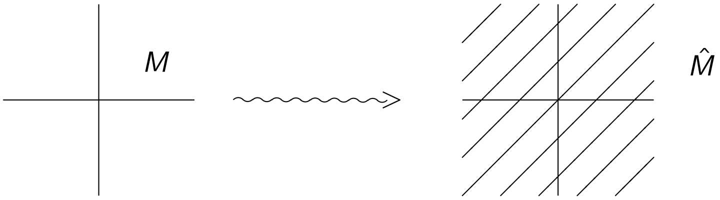

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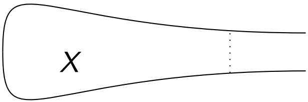

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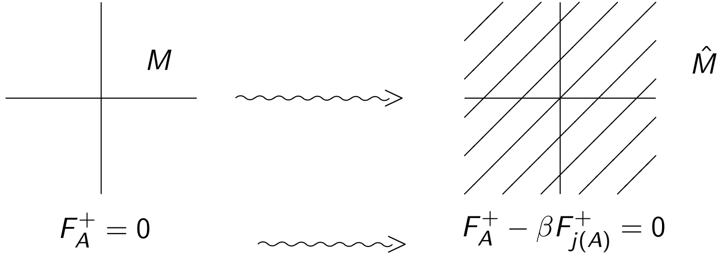

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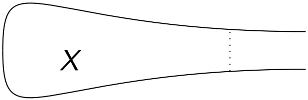
- If A is ASD (and sufficiently close to a reference connection), then $\beta F_{j(A)}^+ = 0$.



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Then we consider the space:

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When these choices are relevant, write

$$\hat{M}(\Gamma, A', \mathcal{T}), \qquad \hat{M}(\Gamma, A'), \qquad \text{etc.}$$

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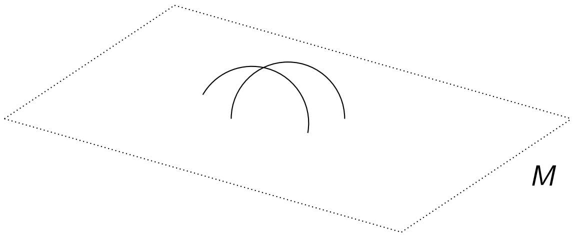

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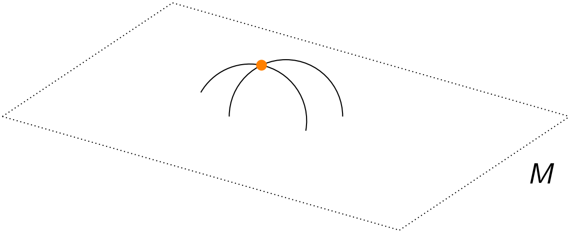

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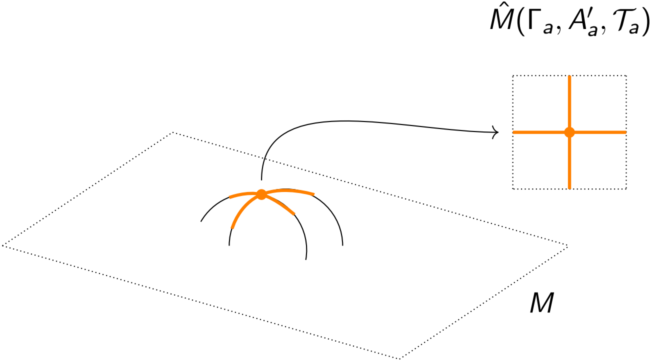

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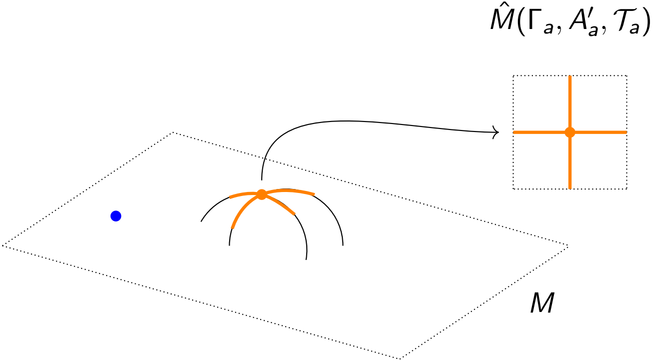

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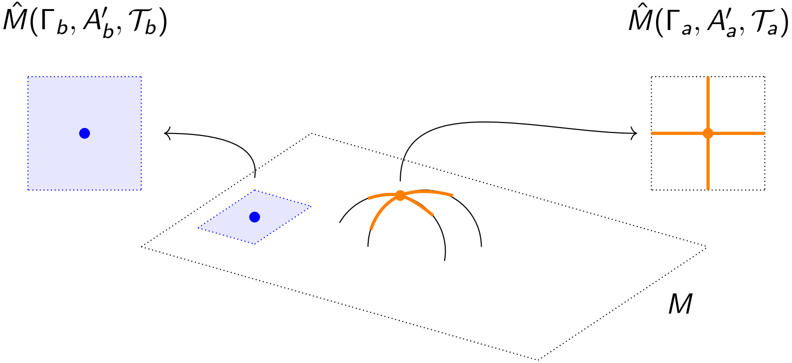

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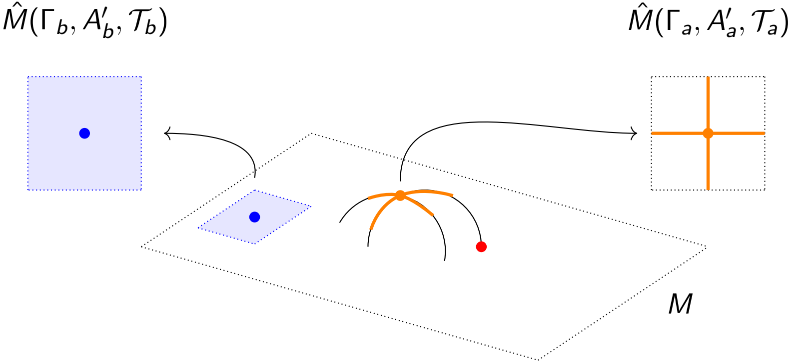

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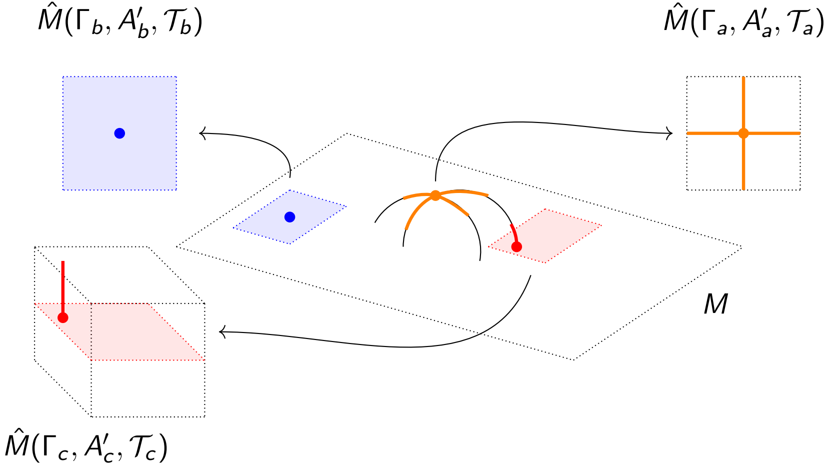

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What is the same? Gluing! (mostly)

Main Results

Theorem (D–Hambleton '20)

Assume $b_2^+(X) = 0$. Then there is an mASD connection on E that is irreducible and regular. This can be chosen to have relative second Chern class arbitrarily close to 1.

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The connection $j(A)$ is ASD for all relevant A. (Observation by Morgan–Mrowka–Ruberman.) □

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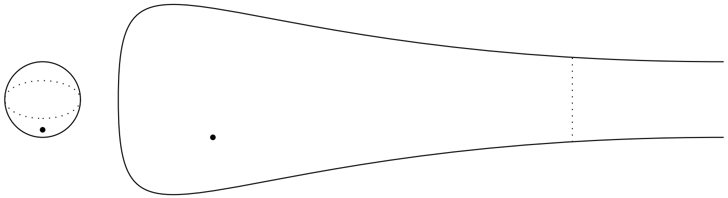

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The linearization of the ASD operator is the map $d^+ : \Omega^1(X, \mathfrak{g}) \to \Omega^+(X, \mathfrak{g})$, and we should extend this to appropriate Sobolev completions, e.g.:

$$d^+ : L^2_1(\Omega^1) \longrightarrow L^2(\Omega^+).$$

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Summary: When $H^1(N) \neq 0$, then the hypothesis that $b^+ = 0$ is generally not enough to use ASD connections alone. However, the space of mASD connections is big enough.

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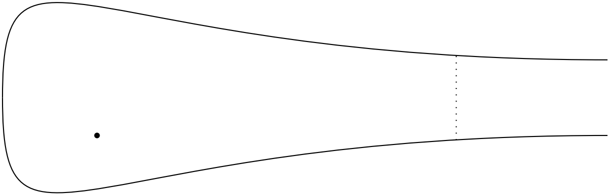

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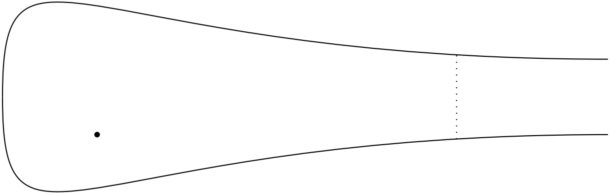

This would give a (partial) compactification.

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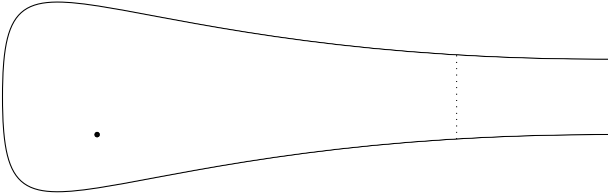

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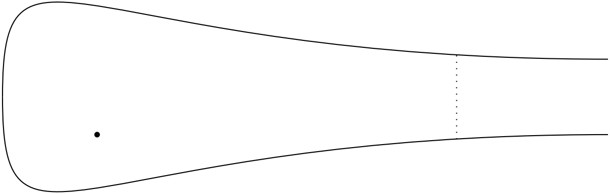

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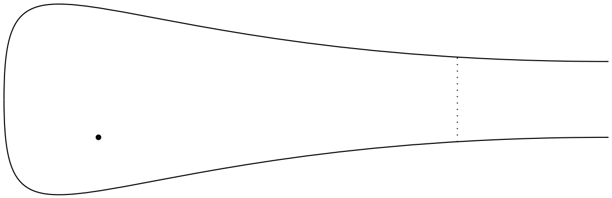

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Resolution: Use the implicit function theorem again.

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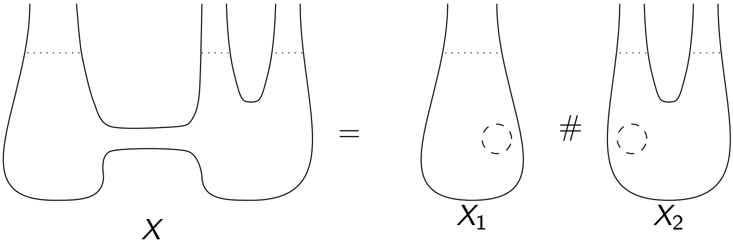

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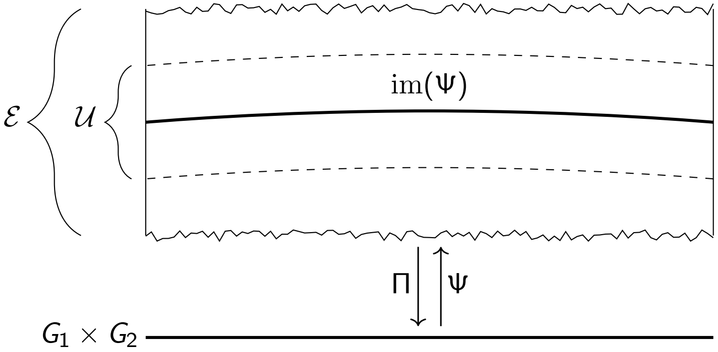

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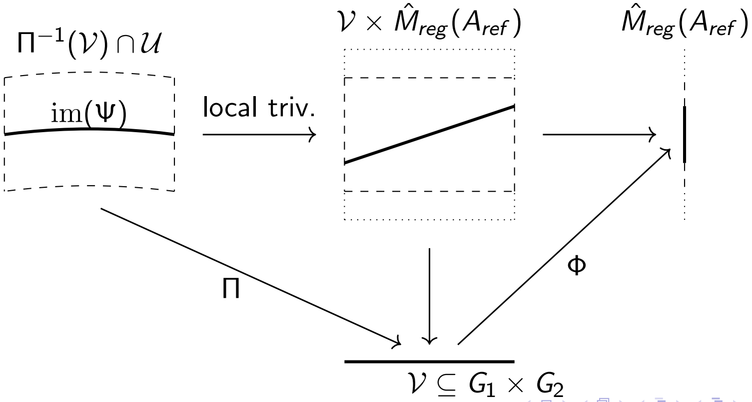

Thank you!