

January 29, 2013



1. Solve the following system of equations using whatever technique you wish. (Remember, you ability to earn partial credit for an incorrect answer depends on my ability to tell what you are doing!)

$$\begin{array}{rcl} 2x+y-z & = & 1 \\ x+z & = & 1 \\ -x+y+z & = & 1 \end{array}$$

$$2x + y - z = 1$$

$$x + z = 1$$

$$-x + y + z = 1$$

$$\begin{vmatrix} 1 & 0 & 1 & | & 1 \\ 2 & 1 & -1 & | & 1 \\ -| & 1 & 1 & | & | & 1 \end{vmatrix} \rightarrow \begin{vmatrix} 1 & 0 & 1 & | & 1 \\ 0 & 1 & 3 & -1 \\ 0 & 1 & 2 & 2 \end{vmatrix} \rightarrow \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & -3 \\ 0 & 0 & 5 \end{vmatrix}$$



2. Put the following matrix in reduced row echelon form.

$$\begin{pmatrix}
2 & -1 & -2 & -1 & -1 \\
-1 & 0 & 2 & 2 & -1 \\
0 & 2 & -4 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
2 & -1 & -2 & -1 & -1 \\
-1 & 0 & 2 & 2 & -1 \\
0 & 2 & -4 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
2 & -1 & -2 & -1 & -1 \\
-1 & 0 & 2 & 2 & -1 \\
0 & 2 & -4 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
1 & 0 & -2 & -2 & 1 \\
0 & 1 & -2 & -3 & 13
\end{pmatrix}$$

$$\begin{pmatrix}
1 & 0 & -2 & -2 & 1 \\
0 & 1 & -2 & -3 & 13
\end{pmatrix}$$

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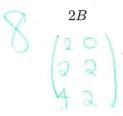
$$\begin{pmatrix}
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0 & 1 & -2 & -3 & 13
\end{pmatrix}$$

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1 & 0 & -2 & -2 & 1 \\
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\end{pmatrix}$$

$$\begin{pmatrix}
1 & 0 & -2 & -2 & 1 \\
0 & 1 & -2 & -3 & 13
\end{pmatrix}$$

$$\begin{pmatrix}
1 & 0 & -2 & -2 & 1 \\
0 & 1 & -2 & -3$$

- a) How many free variables does the corresponding homogeneous system of equations have?
- 3. Given  $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{pmatrix}$  and  $B = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 2 & 1 \end{pmatrix}$  compute the following or say why it is impossible to



$$\begin{pmatrix}
AB \\
5 & 3 \\
3 & 2 \\
4 & 2
\end{pmatrix}$$

$$BB^{T}$$

$$\begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 5 \end{pmatrix}$$

$$AB+B$$

4. Solve the following system for the vector 
$$\overrightarrow{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix} \overrightarrow{x} + \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix} \overrightarrow{X} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix}$$

$$\begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix} \stackrel{?}{x} = \begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \stackrel{?}{x} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \qquad 3 \stackrel{?}{x} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} \qquad \stackrel{?}{x} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

5. Compute the inverse of the following matrices if possible. If it is not possible, say explicitly why it

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 2 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 9 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 5 & 7 & 9 \end{pmatrix}$$

$$\left(\begin{array}{ccc} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 5 & 7 & 9 \end{array}\right)$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 2 & 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

6. Compute the determinate of the following matrices: In each case, what does your answer imply

$$\begin{pmatrix} 2 & 0 & 0 \\ 3 & 1 & 0 \\ 7 & 8 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 & 0 \\ 3 & 1 & 0 \\ 7 & 8 & 2 \end{pmatrix} \qquad \begin{pmatrix} 1 & -1 & 2 \\ 2 & 1 & 1 \\ -1 & -1 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 2 & -1 & 0 \\ -1 & 0 & 1 & 2 \\ 0 & 0 & 2 & 0 \\ 1 & 2 & -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix}
1 & 2 & -1 & 0 \\
-1 & 0 & 1 & 2 \\
0 & 0 & 2 & 0 \\
1 & 2 & -1 & 0
\end{pmatrix}$$

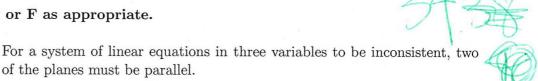
$$\frac{2}{2} \cdot \det \begin{pmatrix} \frac{1}{2} & \frac{2}{2} & -1 \\ 0 & \frac{1}{2} & \frac{2}{2} \end{pmatrix}$$

1 det ( 
$$\frac{1}{4}$$
 )  $=$  (-1) det (  $\frac{2}{4}$  )  $+$  2 det (  $\frac{2}{4}$  )

$$= 2(-2) \det \left( \begin{array}{c} 1 & 2 \\ 12 \end{array} \right) = 1$$

$$= 2+3+2(-2+1)=3$$

## 7. True/False, circle T or F as appropriate. a) $\mathbf{T}$



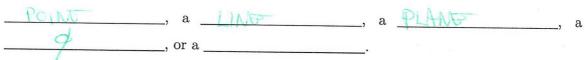
b) A system of three equations in two unknowns is always inconsistent. T

of the planes must be parallel.

- c) It is possible for a homogeneous system of equations to be inconsistent.
- d) T  $\mathbf{F}$ The same coefficient matrix can correspond to either an inconsistent or a redundant system depending on the constant terms.
- e) The solution to a system of linear equations corresponding to an augmented matrix is preserved under column operations.
- f)  $\mathbf{F}$ There is a natural extension of the definition of inverse matrix to both non-square matrices and to non-invertable matrices.
- $(AB)^{-1} = A^{-1}B^{-1}$ g)  ${f T}$
- h)  $\mathbf{T}$ If the determinant of a matrix A is 3, then the determinant of the matrix 2A is 6.

## 8. Fill in the blank.

a) The solution set to a system of three equations in three unknowns corresponds to one of the following geometric objects (list all of the possibilities, there may be more blanks than necessary).



- b) Our three types of elementary row operations are MUST ROW BY NEW 7000, and REPITER
- c) Their corresponding elementary matrices look like



can think elementary of in matrices the following two ways: as and as

	<b>e</b> )	An $n \times n$ matrix is invertable if and only if any one of the following equivalent conditions hold
		RREF IS IDENTIFY
		SOLUTIONS TO SUSTIM OF ER ARE UNIQUE
	f)	The most important theorem about determinants states that
		det(AB)=detAdetB
9.	If i	press the following matrix as the product of elementary matrices or say why this is impossible. It is not possible, what is the closest you can come to this? (This matrix might look familiar mearlier in this exam.)
		$\begin{pmatrix} 2 & 1 & -1 \\ 1 & 0 & 1 \\ -1 & 1 & 1 \end{pmatrix}$
		(-1 1 1) REVERSE =
-1	VE	$\chi^{4} + \chi^{4} + \chi^{4$
(11)		(13)(11)(11)(11)(11)(11)(21)(11)
10.	ar fa	ONUS: We can perform Elementary Column Operations on a matrix A by right multiplying by an elementary matrix (that is, computing AE for E one of our elementary matrices). Given this act, what effect should column operations have on the determinant of a matrix? (Feel free to list his effect in terms of the individual types of elementary column operations.)
		OLES SANGE THING
		AS ROW OPORTRANS
		SINCE det(AE) = detAdetE
		-1 or c or 1
		THE COR COR 1
	<u> </u>	
	Sc	
A=	K	