238 Chapter 1 Test

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January 29, 2013 88 **Name:** KEY

By printing my name I pledge to uphold the honor code.

1. Solve the following system of equations using whatever technique you wish. (Remember, you ability to earn partial credit for an incorrect answer depends on my ability to tell what you are doing!)

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$$\begin{aligned} 2x + y - z &= 1 \\ x + z &= 1 \\ -x + y + z &= 1 \end{aligned}$$

$\left(\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 1 & 0 & 1 & 1 \\ -1 & 1 & 1 & 1 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 2 & 1 & -1 & 1 \\ -1 & 1 & 1 & 1 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & 2 & 2 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 5 & 3 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & 3/5 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2/5 \\ 0 & 1 & 0 & 4/5 \\ 0 & 0 & 1 & 3/5 \end{array}\right)$

$x = \frac{2}{5}$, $y = \frac{4}{5}$, $z = 3/5$

A^{-1}: $\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/5 & 2/5 & -1/5 \\ 0 & 1 & 0 & 2/5 & -1/5 & 3/5 \\ 0 & 0 & 1 & -1/5 & 3/5 & 1/5 \end{array}\right)$

$y = 2/5$, $y = 4/5$, $z = 3/5$

2. Put the following matrix in reduced row echelon form.

$$\begin{pmatrix} 2 & -1 & -2 & -1 & -1 \\ -1 & 0 & 2 & 2 & -1 \\ 0 & 2 & -4 & 0 & 0 \end{pmatrix}$$

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ECHLON FORM 3½

$\begin{pmatrix} 1 & 0 & -2 & 0 & -1/3 \\ 0 & 1 & -2 & 0 & -6 \\ 0 & 0 & 0 & 1 & -1/3 \end{pmatrix}$

a) How many *free variables* does the corresponding homogeneous system of equations have?

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3. Given $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 2 & 1 \end{pmatrix}$ compute the following or say why it is impossible to compute

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$2B$	AB	BB^T	$AB + B$
$\begin{pmatrix} 2 & 0 \\ 2 & 2 \\ 4 & 2 \end{pmatrix}$	$\begin{pmatrix} 5 & 3 \\ 3 & 2 \\ 4 & 2 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 5 \end{pmatrix}$	$\begin{pmatrix} 6 & 3 \\ 4 & 3 \\ 6 & 3 \end{pmatrix}$

4. Solve the following system for the vector $\vec{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

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$$\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix} \vec{x} + \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix} \vec{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ 1/2

MATRIX MULT IN WRONG DIRECTION =3

$\begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix} \vec{x} = \begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \vec{x} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$ $\quad 3\vec{x} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$ $\quad \vec{x} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

5. Compute the inverse of the following matrices if possible. If it is not possible, say explicitly why it is not.

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$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 2 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 9 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 5 & 7 & 9 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{pmatrix}$$

NOT SQUARE!

$\begin{pmatrix} 1 & & \\ & 1/3 & \\ & & 1/9 \end{pmatrix}$

$\begin{pmatrix} 1 & & -2 \\ & 1 & \\ & & 1 \end{pmatrix}$ det=1

det=0 NOT INVERTABLE.

45+60+84 −75−72−42

75, 72, 42: 189 — 45, 60, 84: 18.9

WORK. 1+2−3 det=0

$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 2 & 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right) = \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right)$

6. Compute the determinate of the following matrices: In each case, what does your answer imply about the matrix?

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$$\begin{pmatrix} 2 & 0 & 0 \\ 3 & 1 & 0 \\ 7 & 8 & 2 \end{pmatrix} \quad \begin{pmatrix} 1 & -1 & 2 \\ 2 & 1 & 1 \\ -1 & -1 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 2 & -1 & 0 \\ -1 & 0 & 1 & 2 \\ 0 & 0 & 2 & 0 \\ 1 & 2 & -1 & 0 \end{pmatrix}$$

det=4 MATRIX IS INVERTABLE.

1+1−4 −((−2)+−2−1) −2+5=3 MATRIX IS INVERTABLE!

$2 \cdot \det\begin{pmatrix} 1 & 2 & -1 \\ 0 & 0 & 2 \\ 1 & 2 & -1 \end{pmatrix}$

$1 \det\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} - (-1)\det\begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix} + 2\det\begin{pmatrix} 2 & 1 \\ -1 & -1 \end{pmatrix}$

$= 2+3+2(-2+1) = 3$

$= 2(-2)\det\begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} = 0$

⇒ MATRIX ISN'T INVERTABLE!

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7. True/False, circle T or F as appropriate.

a) T **F** For a system of linear equations in three variables to be inconsistent, two of the planes must be parallel.

b) T **F** A system of three equations in two unknowns is always inconsistent.

c) T **F** It is possible for a homogeneous system of equations to be inconsistent.

d) **T** F The same coefficient matrix can correspond to either an inconsistent or a redundant system depending on the constant terms.

e) T **F** The solution to a system of linear equations corresponding to an augmented matrix is preserved under column operations.

f) **T** F There is a natural extension of the definition of inverse matrix to both non-square matrices and to non-invertable matrices.

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g) T **F** $(AB)^{-1} = A^{-1}B^{-1}$

h) T **F** If the determinant of a matrix A is 3, then the determinant of the matrix $2A$ is 6.

8. Fill in the blank.

a) The solution set to a system of three equations in three unknowns corresponds to one of the following geometric objects (list all of the possibilities, there may be more blanks than necessary).

4 POINT, a LINE, a PLANE, a ∅, or a ________.

6 **b)** Our three types of elementary row operations are SWITCH 2 ROWS, MULT ROW BY NON-ZERO CONSTANT, and REPLACE ROW BY ITSELF + MULT. OF ANOTHER ROW

c) Their corresponding elementary matrices look like

6 $$E_1 = \begin{pmatrix} \quad \end{pmatrix}, E_2 = \begin{pmatrix} \quad \end{pmatrix}, \text{ and } E_3 = \begin{pmatrix} \quad \end{pmatrix}$$

d) We can think of elementary matrices in the following two ways: as IDENTITY MATRIX AFTER ROW OP. and as THE MATRIX THAT PERFORMS ROW OPS VIA LEFT MULT.

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e) An $n \times n$ matrix is invertable if and only if any one of the following equivalent conditions hold

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det ≠ 0

RREF is IDENTITY

SOLUTIONS TO SYSTEM OF EQ. ARE UNIQUE

f) The most important theorem about determinants states that

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det(AB) = det A det B.

9. Express the following matrix as the product of elementary matrices or say why this is impossible. If it is not possible, what is the closest you can come to this? (This matrix might look familiar from earlier in this exam.)

$$\begin{pmatrix} 2 & 1 & -1 \\ 1 & 0 & 1 \\ -1 & 1 & 1 \end{pmatrix}$$

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NOT REVERSE ORDER = −1

$= A^{-1}$

10. BONUS: We can perform Elementary Column Operations on a matrix A by *right* multiplying by an elementary matrix (that is, computing AE for E one of our elementary matrices). Given this fact, what effect should column operations have on the determinant of a matrix? (Feel free to list this effect in terms of the individual types of elementary column operations.)

DOES SAME THING AS ROW OPERATIONS

SINCE det(AE) = det A det E

−1 OR c OR 1 DEPENDING ON TYPE!

SO

A = ...