238 Chapter 4 Test

March 26, 2013 **Name:** _____

By printing my name I pledge to uphold the honor code.

Unless otherwise specified, *solution* refers to a real valued function.

1. Fill in the blank.

a) All solutions to a linear ODE are of the form _____

where _____ is a solution to _____ and _____ is a solution

to _____.

b) The solutions to an _____th order _____ ODE form an n-dimensional vector space.

c) Our technique for solving homogeneous linear ODE's only applies to ones with

_____ and relies on finding the _____

of the _____.

2. Determine the general solution for the following differential equations.

a) $y''' - y'' + y' - y = 0$

b) $y'' - 4y' + 4y = e^{-x}$

3. Solve the following initial value problem.

$$y'' - 2y' + 2y = \cos x,\ y(0) = 1,\ y'(0) = 2$$

4. Non constant coefficient ODEs.

a) Show that the given set of functions forms a fundamental solution set to the homogeneous ODE.

$$\left\{\frac{1}{x}, x\right\}, x^2y'' + xy' - y = 0$$

b) Use the information from part a to find a particular solution to the following non-homogeneous ODE.

$$x^2y'' + xy' - y = \frac{1}{x}$$

5. True/False, circle T or F as appropriate.

a) **T** **F** Phase portraits are not useful for n^{th} order ODEs because the phase portrait would have to live in $\mathbb{R}^n$ because of all the data necessary for an IVP.

b) **T** **F** The sum of two solutions to a general homogeneous ODE is still a solution to that equation.

c) **T** **F** If λ is a root of the characteristic polynomial for linear constant coefficient homogeneous ODE, then $e^{\lambda x}$ is always a solution to the (homogeneous) ODE, even if it is not a real valued function.

d) **T** **F** If a complex valued function is a solution to a homogeneous linear ODE, then both is real part and its imaginary part are individually solutions.

e) **T** **F** The above is false for non-linear homogeneous ODEs.

6. More fill in the blanks.

a) An initial value problem for an nth order ODE involves giving _____ extra pieces of information, usually in the form of _____.

b) The characteristic polynomial for a constant coefficient homogeneous linear ODE is extracted by substituting $y =$_____ into the ODE.

c) Given the non-homogeneous linear differential equation $y'' + 3y' + 5y = e^x \cos x$ the method of undetermined coefficients would give our first guess for

$y_P =$ _____.

d) The non-homogeneous linear ODE $y'' + 3y' + 5y = \sec x$ is not a good choice for the method of undetermined coefficients because _____.

e) The method of variation of parameters involves setting

$y_P =$_____ where _____ and solving for _____ by making the simplifying assumption that

_____.