238 Chapter 5 Test

April 16, 2013 **Name:** ______________________________

By printing my name I pledge to uphold the honor code.

1. Fill in the blank.

a) A function $T : V \to W$ between vector spaces V and W is a linear transformation if the following two equations are satisfied: __________________________ holds for all ________________ and __________________________ holds for all ________________________.

b) A linear transformation $T : V \to W$ is completely determined by its value on ____.

c) Eigenvalues for a matrix A are found by computing __________________________ while eigenspaces are found by computing __________________________________.

2. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $T\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right) = \begin{pmatrix} x - y + z \\ 3y \end{pmatrix}$.

a) Prove that T is a linear transformation using the definition.

b) Let α be the standard basis for $\mathbb{R}^3$ and β be the standard basis for $\mathbb{R}^2$. Find $[T]_\alpha^\beta$.

3. Given $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T\left(\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$ and $T\left(\begin{pmatrix} -1 \\ 2 \end{pmatrix}\right) = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$

a) Given the matrix for T in terms of the standard basis.

$$[T] =$$

b) Give a matrix $[T]_\beta^\beta$ such that $[T]_\beta^\beta$ is in Jordan canonical form for some basis β. You do not need to find β.

$$[T]_\beta^\beta =$$

4. The characteristic polynomial of the matrix $A = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}$ is $(\lambda + 1)^2(\lambda - 8)$.

a) Compute all of the eigenspaces for the matrix A. You may not need all of the blanks.

$E_{\_\_\_} = \{ \qquad\qquad\qquad\qquad \}$.

$E_{\_\_\_} = \{ \qquad\qquad\qquad\qquad \}$.

$E_{\_\_\_} = \{ \qquad\qquad\qquad\qquad \}$.

b) Find P and D such that D is diagonal and $A = PDP^{-1}$.

$$D =$$

$$P =$$

5. True/False, circle T or F as appropriate. *If you have time after you have finished the rest of the exam* you may go back and explain your answers for possible partial credit/extra credit.

a) **T** **F** If $T(\mathbf{0}_V) \neq \mathbf{0}_W$, then T is not a linear transformation.

b) **T** **F** The definition of matrix multiplication from Chapter 1 comes from what we would need for composition of linear transformations to be multiplication of the cooresponding matrices.

c) **T** **F** Every matrix has a unique Jordan canonical form (up to reordering the jordan blocks).

d) **T** **F** A two by two matrix with repeated eigenvalues is either already diagonal or not diagionalizable.

e) **T** **F** $\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ is an eigenvector for $\begin{pmatrix} 1 & -1 & 2 & 1 \\ -6 & 9 & -1 & 1 \\ 1 & 2 & -3 & 3 \\ 4 & -2 & 1 & 0 \end{pmatrix}$.

6. More fill in the blanks.

a) A matrix is diagionalizable if and only if

____.

b) The dimension of the eigenspace $E_\lambda \leq$ ______________________________.

c) Describe one way the diagional form of a matrix can be useful.

____.

7. Describe the linear transformation from problem 3 in terms of what the transformation does to $\mathbb{R}^2$ *geometrically.*