Math 300 Linear Independence Problems

1. $\mathbb{R}^n$ revisited

a) Suppose that $v_1, v_2, ..., v_k$ are vectors in $\mathbb{R}^n$. Explain how we can tell from an echelon form of the matrix $A - (v_1, v_2, ..., v_k)$ whether $v_1, v_2, ..., v_k$ form a linearly independent set. What are you looking for in the matrix to tell you whether this set of vectors is linearly independent or linearly dependent? Why does this feature of the matrix let you make this conclusion?

b) Explain why a set of more than n vectors in $\mathbb{R}^n$ must be linearly dependent.

c) Does this mean that every set n or fewer vectors in $\mathbb{R}^n$ is linearly independent?

2. Suppose $\{v_1, v_2\}$ is a linearly independent set in $\mathbb{R}^3$.

a) What geometric object do the linear combinations of v_1 and v_2 determine?

b) If v_3 is a third vector in $\mathbb{R}^3$, what geometric condition about v_3 must hold in order for $\{v_1, v_2, v_3\}$ to be a linearly independent set?