Math 300 Section 1.1 Additional Problems

1. Let U, V and W vectors in $\mathbb{R}^2$ be as follows: $U = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $V = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ and $W = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$

a) Sketch U, V, and W on the plane.

b) Sketch $U + V$.

c) Sketch $2U, -3V$, and $\frac{1}{2}W$.

d) Sketch $-3V + \frac{1}{2}W$.

2. Show that all of the properties of a vector space hold for $\mathcal{F}(\mathbb{R})$ where $\mathcal{F}(\mathbb{R})$ is defined as in Theorem 1.2.

3. In each of the following, determine whether the indicated addition and scalar multiplication on ordered pairs of real numbers yields a vector space. If a property holds, show/prove that it does. If a property does not hold, find a counterexample (and show its a counterexample).

a) $(x_1, y_1) \oplus (x_2, y_2) = (x_1, y_2), \quad k \odot (x, y) = (kx, ky)$

b) $(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2), \quad k \odot (x, y) = (k + x, k + y)$

c) $(x_1, y_1) \oplus (x_2, y_2) = (x_1 + y_2, x_2 + y_1), \quad k \odot (x, y) = (kx, ky)$

4. Determine whether the indicated addition and scalar multiplication on ordered pairs of **positive** real numbers yields a vector space. If a property holds, show/prove that it does. If a property doesn't hold, find a counterexample (and show it's a counterexample).

$(x_1, y_1) \oplus (x_2, y_2) = (x_1x_2, y_1y_2), \quad k \odot (x, y) = (x^k, y^k).$

5. Prove the following statements about a vector space $\mathcal{V}$.

a) A zero vector of $\mathcal{V}$ is unique.

b) For any vector $V \in \mathcal{V}, 0 \cdot V = \mathbf{0}$, where the zero on the left is the number 0 and the $\mathbf{0}$ on the right is the zero vector.