Math 300 Section 2.3 Additional Problems

1. Suppose a 5×8 matrix A has exactly 5 pivot columns.

a) What is rank(A)?
b) What is null(A)?
c) Is $CS(A) = \mathbb{R}^5$?
d) Is $NS(A) = \mathbb{R}^3$?
e) Does $A\overrightarrow{X} = \overrightarrow{B}$ have at least one solution for each $\overrightarrow{B} \in \mathbb{R}^5$?
f) If $\overrightarrow{B} \in \mathbb{R}^5$ and $A\overrightarrow{X} = \overrightarrow{B}$ has at least one solution, how many linearly independent spanning vectors appear in the general solution of $A\overrightarrow{X} = \overrightarrow{B}$?
g) Are the columns of A linearly independent?
h) Suppose $\overrightarrow{X}_1, \overrightarrow{X}_2 \in NS(A)$. Can $\overrightarrow{X}_1$ and $\overrightarrow{X}_2$ span $NS(A)$?

2. Is there a 5×8 matrix A with rank$(A) = 2$? Why/why not?

3. Suppose A is a 7×4 matrix with rank$(A) = 4$.

a) How many zero rows are in the row reduced echelon form of A?
b) What is dim$(CS(A))$?
c) Do the columns of A span $\mathbb{R}^7$?
d) What is null(A)?
e) Are any of the coumns of A linear combinations of the other columns of A?
f) Does $A\overrightarrow{X} = \overrightarrow{B}$ have at most one solution for each $B \in \mathbb{R}^7$?
g) Does $A\overrightarrow{X} = \overrightarrow{B}$ have exactly one solution for each $B \in \mathbb{R}^7$?
h) Are the 7 original rows of A linearly independent?

4. Suppose A is a 3×6 matrix and rank$(A) = 2$.

a) How many non-pivot columns does A have?
b) How many nonzero rows are in an echelon form of A?
c) Does $A\overrightarrow{X} = \overrightarrow{0}$ have non-trivial solutions?
d) What is dim$(NS(A))$?
e) If $\{\overrightarrow{X}_1, \overrightarrow{X}_2, \overrightarrow{X}_3, \overrightarrow{X}_4\}$ are linearly independent vectors in $NS(A)$, do $\overrightarrow{X}_1, \overrightarrow{X}_2, \overrightarrow{X}_3$, and $\overrightarrow{X}_4$ span $NS(A)$?
f) If $\{\overrightarrow{Y}_1, \overrightarrow{Y}_2, \overrightarrow{Y}_3, \overrightarrow{Y}_4\}$ is a spanning set for $CS(A)$, is $\{\overrightarrow{Y}_1, \overrightarrow{Y}_2, \overrightarrow{Y}_3, \overrightarrow{Y}_4\}$ linearly independent?

5. An engineer solves a system of 10 linear equations in 12 unknowns and finds that the general solution of the system contains four linearly independent spanning vectors. Can the engineer be certain that, if the right sides of the equations are changed, the new system (with the same coefficients as before) will have at least one solution? Explain.

6. Suppose a system of five linear equations in seven unknowns has a solution with two free variables. Is it possible to change some constants on the equations right sides to make the new system inconsistent? Explain.

7. A homogeneous system of eleven linear equations in eight unknowns has two fixed solutions that are not multiples of each other, and all other solutions are linear combinations of these two solutions. Can the set of all solutions to this system be described with fewer than 11 homogeneous linear equations? If so, how many are needed? Explain.