Additional Suggested HW for Section 3.1, part 2

1. Let $T : \mathbb{P}_1 \to \mathbb{P}_2$ be defined by $T(ax+b) = 2ax^2 + bx + (a+b)$.
   (a) Show that T is a linear transformation.
   (b) Describe the nullspace of T, $NS(T)$, and the image of T, $T(\mathbb{P}_1)$
   (c) Is T one-to-one? Is T onto?

2. Let $T : \mathbb{R}^4 \to \mathbb{R}^2$ be defined by $T\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} x + 2w \\ y + z + w \end{bmatrix}$.
   (a) Use algebra to find a matrix A such that $T(X) = AX$ for all $X \in \mathbb{R}^4$. Why does this show that T is a linear transformation?
   (b) Find bases for $NS(T)$ and $T(\mathbb{R}^4)$.
   (c) Is T one-to-one? Is T onto?

3. Let $A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & 5 & -3 \\ -3 & 2 & 4 \end{bmatrix}$. Let $T_A : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T_A(X) = AX$.
   Is T_A one-to-one? Is T_A onto? (Notice: I didn't ask for bases for any spaces, so you can use dimension or equivalent results.)

4. Let $A = \begin{bmatrix} 1 & 3 & 4 \\ -2 & -5 & -3 \\ 1 & 4 & 9 \end{bmatrix}$. Let $T_A : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T_A(X) = AX$.
   Is T_A one-to-one? Is T_A onto? (Notice: I didn't ask for bases for any spaces, so you can use dimension or equivalent results.)