Additional HW for Section 3.3

1. Determine whether the following statements are true or false. Justify your answers.
   (a) If A is an invertible matrix, then A^{-1} and A^2 are invertible.
   (b) If A and B are invertible $n \times n$ matrices, then $A^{-1}B^{-1}$ is the inverse of AB.
   (c) If A and B are invertible $n \times n$ matrices, then $A + B$ is invertible.
   (d) If A is an invertible $n \times n$ matrix, then the equation $AX = B$ has at least one solution for each B in $\mathbb{R}^n$.
   (e) If A is an invertible $n \times n$ matrix and B and C are $n \times p$ matrices with $AB = AC$, then $B = C$.
   (f) If A can be row reduced to the identity matrix, then A must be invertible.
   (g) If A is an invertible $n \times n$ matrix, then $AX = \mathbf{0}$ has nontrivial solutions.
   (h) If A is an $n \times n$ matrix and the columns of A span $\mathbb{R}^n$, then A is invertible.
   (i) If A and B are $n \times n$ matrices such that $AB = 0$ and B is invertible, then $A = 0$.

2. Explain why a square matrix containing a zero row or a zero column cannot be invertible.

3. Suppose A and B are $n \times n$ matrices and $C = AB$ is invertible. Prove that both A and B are invertible and find A^{-1} and B^{-1}. (Hint: Find a formula for A^{-1} that involves C^{-1} and B. Similar hint for B^{-1}.)

4. Suppose A is an $n \times n$ matrix and there exists a matrix B such that $BA = I$. Prove that $NS(A) = \{\vec{0}\}$. Why does this show that A is invertible?

5. Suppose A is an $n \times n$ matrix and there exists a matrix B such that $AB = I$. Prove that, for each $Y \in \mathbb{R}^n$, $AX = Y$ has at least one solution. Why does this show that A is invertible.

6. Prove that the inverse of an invertible matrix is unique.