Additional Suggested HW for Section 4.2

1. Use determinants to answer the following questions:

   (a) Do $\begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 8 \end{bmatrix}$ form a basis for $\mathbb{R}^3$?

   (b) Are $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$ linearly independent?

   (c) Let $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 0 \\ 1 & 5 & 2 \end{bmatrix}$. Is $NS(A) = \{0\}$? Is multiplication by A one-to-one?

   (d) Let $B = \begin{bmatrix} 2 & -1 & 3 & 5 \\ 2 & 0 & 1 & 0 \\ 6 & 1 & 3 & 4 \\ -7 & 3 & -2 & 8 \end{bmatrix}$. Do the columns of B span $\mathbb{R}^4$? Is multiplication by B an onto transformation onto $\mathbb{R}^4$?

2. Why can we not use determinants to decide whether $\begin{bmatrix} 4 \\ -1 \\ 2 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -5 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -4 \\ 9 \\ 0 \end{bmatrix}$ are linearly independent?