Math 300 Spans Problems

1. Is $\begin{bmatrix} 3 & 5 \\ 3 & 6 \end{bmatrix}$ in Span $\left\{ \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \right\}$?

2. Is $3x^2 \in \text{Span}\{x^2 - x, x^2 + x + 1, x^2 - 1\}$?

3. Determine whether $x^2 - 4, x^2 + 4$, and $x^2 + x$ span $\mathbb{P}_2$ where $\mathbb{P}_2$ is the vector space of all polynials of degree less than or equal to 2.

4. $\mathbb{R}^n$

a) Suppose that $v_1, v_2, ..., v_k$ are vectors in $\mathbb{R}^n$. Explain how you can tell from an echelon form of the matrix $A = (v_1, v_2, ..., v_k)$ whether $v_1, v_2, ..., v_k$ span $\mathbb{R}^n$. What are you looking for in a matrix to tell you whether the vectors span or not? Why does this feature of the matrix let you make this conclusion?

b) Explain why a set of fewer than n vectors in $\mathbb{R}^n$ cannot span $\mathbb{R}^n$.

c) Does this mean that every set of n or more vectors in $\mathbb{R}^n$ spans $\mathbb{R}^n$?