411 REAL ANALYSIS II WEEKLY QUIZ 1 (MATH 410 REVIEW)

January 28-29, 2016 **Name:** ______________________

By printing my name I pledge to uphold the honor code.

No books, notes, internet, etc. This should take about a half hour.

REMBER TO SIGN IN!

1. $\mathbb{R}^1$

a) Please define the infemium of a set. Give the two conditions that an infemium needs to satisfy in terms of inequalities. When do supremiums/infemiums exist? (We will *definitely* need this for 411!)

b) Please state the Nested Interval Property and its extension to deal with intervals whose lengths go to zero. Give examples of this property in use.

2. Sequences

a) A metric space X is .... What are our three main examples of metric spaces and what purpose do they serve?

b) A sequence $\{a_n\}_{n=1}^{\infty}$ in a metric space X converges to $a \in X$ if ...

c) Define a Cauchy sequence and list some theorems involving sequence convergence and Cauchy sequences. What sequences are Cauchy in your three examples from 2a?

d) Define a complete metric space and list various completions of $\mathbb{Q}$ their advantages and disadvantages.

3. Topology

a) Please define an open set in a metric topology.

b) Please define a compact set. Explain why neither $\mathbb{R}$ nor $(0, 1]$ are not compact using your definition.

c) Please state some theorems about compact sets, be sure to specify in what generality each is true.

4. Continuity. Let $f : X \to Y$ where X and Y are metric spaces.

a) Give the definition of $\lim_{x \to c} f(x) = L$ and specify how it relates to sequence definition of limit in both X and Y.

b) Give two different definitions of a continuous function. Why should these two defintions be related?

c) Give the major theorem that is key to single variable calculus working. what is the general version of this theorem and why is it not very hard to prove?

REMEMBER TO PUT YOUR COMPLETED QUIZ IN THE CORRECT ENVELOPE AND SIGN OUT!