411 REAL ANALYSIS II WEEKLY QUIZ 5

March 30-31, 2016 **Name:** _____

By printing my name I pledge to uphold the honor code.

No books, notes, internet, etc. This should take about 50 minutes.

REMBER TO SIGN IN!

1. Measure theory

a) What are the three properties that define a σ-algebra and what is the purpose of a σ-algebra? Please give 4 examples of σ-algebras.

b) Please define (on $\mathbb{R}^1$) the Lebesgue outer measure $\lambda^* :? \to \mathbb{R}^{\geq 0} \cup \{\infty\}$. What is "?"

c) How do we go from the Lebesgue outer measure to the Lebesgue measure on $\mathbb{R}^1$. Please be as specific as possible.

2. Riemann-Lebesgue theorem

a) Please state the Riemann-Lebesgue theorem.

b) What are the main ideas in its proof?

3. FTC

a) Please state both parts of the Fundamental Theorem of Calculus.

b) What are the ideas behind the proofs of your theorems from a)?

4. Sequences of functions

a) Define two different notions of convergence of sequences of functions.

b) Illustrate using words and picture which of the two notions from part a) is preferred and why. Namely, give an example where the less good notion goes wrong and illustrate why the better notion has no such issues.