411 REAL ANALYSIS II WEEKLY QUIZ 6

April 8, 2016 **Name:** ______________________________

By printing my name I pledge to uphold the honor code.

No books, notes, internet, etc. This should take about 30 minutes.

REMBER TO SIGN IN!

1. Pointwise vs Uniform Convergence. Let $f_n : X \to Y$ for $n \in \mathbb{N}$ and Y a complete metric space.

a) Please define pointwise and uniform convergence for the sequence $\{f_n\}_{n=1}^{\infty}$.

b) What is properties of $f_n : X \to Y$ if Y is a complete metric space are preserved under pointwise continuity? (Yes, there is at least one!!) What statement would we need to prove to show that uniform convergence preserves continuity? Approximately why should this hold?

c) Please outline at least one application of part b) to measure theory.

2. $\mathcal{C}_Y(X)$

a) What is $\mathcal{C}_Y(X)$? What are our main examples of Y and why?

b) Please define the sup norm on $\mathcal{C}_Y(X)$ where Y is a normed space. What is the induced metric on the space $\mathcal{C}_Y(X)$?

c) What does it mean for a sequence $\{f_n\}_{n=1}^{\infty} \subset \mathcal{C}_Y(X)$ to be Cauchy? Why?

d) How would we go about showing that $\mathcal{C}_{\mathbb{R}}(X)$ is a complete metric space?

e) What does it mean for a subset $F \subset \mathcal{C}_Y(X)$ to be pointwise or uniformly bounded? What was the purpose of those two definitions?