411 REAL ANALYSIS II WEEKLY QUIZ 7

April 15, 2016 **Name:** KEY

By printing my name I pledge to uphold the honor code.

No books, notes, internet, etc. This should take about 20 minutes.

REMBER TO SIGN IN!

2½

1. The point of all this

a) We are trying to find an analog of what theorem on $\mathcal{C}_Y(X)$? Be as specific as possible. HEINE-BOREL THEOREM

IN $\mathbb{R}^n$ UNDER EUCLIDEAN METRIC CPT $\Leftrightarrow$ CLOSED & BOUNDED.
WE WANT ~~[illegible]~~ AN INDUCTIVE NOTION OF COMPACT IN $\mathcal{C}_Y(X)$.

b) What was our first attempt at doing so and why did it fail?

1st ATTEMPT WAS POINTWISE/UNIFORMLY BOUNDED.
FAILED 'CAUSE SOMETHING LIKE $\{\sin nx\}_{n=1}^{\infty}$ ~~ARE ALL~~ IS UNIF. BOUNDED BUT HAS NO CONVERGENT SUBSEQ.

c) What was the tiny amount of progress we could make using only part b)?

WE WERE ABLE TO FIND A ~~[illegible]~~. SUBSEQ OF ANY $\{f_n\} \subset \mathcal{C}_{\mathbb{R}^n}(X)$ THAT CONVERGED ON A COUNTABLE SUBSET OF X.

2. Equicontinuity

a) Please define an equicontinuous family of functions $\mathcal{F}$ where $f \in \mathcal{F}$ is $f : X \to Y$ with X, Y metric spaces and provide an extra sentence and/or illustration of equicontinuity. IF $\forall \varepsilon > 0, \exists \delta > 0$ st. $d_X(x,y) < \delta \Rightarrow d_Y(f(x), f(y)) < \varepsilon$ $\forall f \in \mathcal{F}$

IT'S UNIF. CONTINUITY FOR ALL $f \in \mathcal{F}$ W/ SAME δ/ε PAIR!

b) Equicontinuity comes for free in what circumstances and why?

IF WE HAVE A CAUCHY SEQ $\{f_n\} \subset \mathcal{C}_Y(K)$ K CPT $\Rightarrow \{f_n\}$ IS EQUI-CONTINUOUS.
THIS IS 'CAUSE ALL FCTNS ARE UNIF. CONT & CAUCHY SEQ ALLOWS COMPARISON BETWEEN FCTNS.
IT ALSO COMES FREE W/ FINITE SET OF UNIF. CONT. FCTNS 'CAUSE ~~[illegible]~~ $\forall \varepsilon > 0$, TAKE $\min\{\delta_i\}$.

3. **AA Theorem**

a) Please state the Arzelá Ascoli theorem and give a few sentence synopsis of why we care about it.

IF $\{f_n\} \subset C_{\mathbb{R}^n}(K)$ K CPT IS AN EQUICONTINUOUS POINTWISE BOUNDED FAMILY $\Rightarrow \{f_n\}$ IS UNIF BOUNDED AND $\exists$ CONV. SUBSEQ.

THIS IMPLIES ~~COMPACT~~ $\{f_n\} \subset$ CPT SET IN $C_{\mathbb{R}^n}(K)$

$\frac{1}{2}$

b) Why should it be true?

EQUICONTINUITY ALLOWS US TO COMPARE BETWEEN FCNS IN OUR FAMILY.
K CPT $\Rightarrow \exists$ COUNTABLE DENSE SUBSET
$\Rightarrow \exists$ CONV. SUBSEQ ON COUNT. DENSE SUBSET
K CPT $\Rightarrow$ ~~...~~ f_n UNIF. CONT.
TOGETHER THEY IMPLY THAT SUBSEQ CONV. ON ALL OF K.

4. **Weierstrass Theorem**

a) Please state a version of the Weirestrass Theorem.

$\forall f \in C_{\mathbb{R}}[a,b]$, $\exists \{P_n\} \subset \mathbb{R}[x]$ st. $P_n \to f$ UNIF. ON $[a,b]$.

$\frac{1}{2}$

b) What is one surprising consequence of the theorem?

$|\mathbb{R}| = |C_{\mathbb{R}}[a,b]| = |C_{\mathbb{C}}[a,b]|$

$\forall \varepsilon > 0\ \exists$ P st. $P(0)=0$ & $\||x|-P\| < \varepsilon$ EVEN THOUGH $|x|$ NON-DIFF.
P POLY

$\mathbb{Q}[x]$ COUNTABLE DENSE SUBSET IN $C_{\mathbb{R}}(K)$ $K \subset \mathbb{R}$ CPT.

REMEMBER TO PUT YOUR COMPLETED QUIZ IN THE CORRECT ENVELOPE AND SIGN OUT!