Math 485 Homework 5

1. Prove that the standard three-dimensional representation of the tetrahedral group (the group of twelve rotations carrying a regular tetrahedron to itself) is irreducible as a complex representation.

2. Determine all irreducible representations of a cyclic group C_n.

3. Consider the standard two-dimensional representation of the dihedral group D_n as symmetries of the n-gon. For which values of n is it irreducible as a complex representation?

4. Find the decomposition of the standard two-dimensional rotation representation of the cyclic group of order n into a product of irreducible representations.

5. Prove or disprove: Let χ be a character of a finite group G, and define $\overline{\chi}(g) = \overline{\chi(g)}$. Then $\overline{\chi}$ is also a character of G.

6. Find the dimensions of the irreducible representations of the dihedral groups D_4, D_5, and D_6.

7. Character tables

a) Compare the character tables for the quaternion group and the dihedral group D_4.

b) Determine the character table for D_6.

c) Find the missing rows in the character table below:

	(1)	(3)	(6)	(6)	(8)
	1	a	b	c	d
χ_1	1	1	1	1	1
χ_2	1	1	-1	-1	1
χ_3	3	-1	1	-1	0
χ_4	3	-1	-1	1	0

d) Determine the character table for the groups C_5 (cyclic group of order 5) and D_5.

e) Decompose the restriction of each irreducible character of D_5 into irreducible characters of C_5.

8. Let $G' = G/N$ be a quotient group of a finite group G and let ρ' be an irreducible representation of G'. Prove that the representation of G defined by ρ' is irreducible in two ways: directly, and using our huge theorem.

9. Let χ be the character of a representation ρ of dimension d. Prove that $|\chi(g)| \leq d$ for all $g \in G$, and that if $|\chi(g)| = d$, then $\rho(g) = \zeta I$ for some root of unity ζ.