Information Sheet

Math 485: Special Topics, Representation Theory, Spring 2017, Professor Rebecca Field
Section 1 MWF 12:20–1:10pm, Roop G10

Suggested Text: *Algebra* by Michael Artin
Exam dates:
Big Quiz: February 7, during class
Midterm: week of March 22, evening
Big Quiz 2: April 12, during class time
Final Exam: May 3, 10:30am-12:30pm

Contact information for Professor Field
Office: Roop 114
Phone: 540-746-1231
Email: fieldre@jmu.edu
Webpage: https://educ.jmu.edu/~fieldre/485repth.html

Office Hours: MWF 2:20-3:20pm, Tu 12:00–2:00pm, and by appointment.
You can always make an appointment to see me

The goal of Math 485 is to provide you with an introduction to a subject that is becoming more and more central to mathematics with each passing year: Representation Theory. A *representation* of a group G is any way to write a group as a set of (invertable) matricies (so as a subgroup of some $GL_n(F)$ for F a field). In fact, all groups have representations, and you are already familiar with some, just possibly not in matrix form. Consider the multiplication table of a group G: any line of the table (which represents left multiplication by an element $g \in G$) contains all of the elements of the group, which means that (left) multiplication by g is a *permutation* of G, and any permutation can be written as a matrix consisting of only 0s and 1s. What we just did was give a represnetation for a group of size n as a subgroup of $GL_n(\mathbb{Z}/2\mathbb{Z})$ that will work for *any* finite group.

More concretely, let G be any group and V be a vector space and $GL(V)$ be the set of linear transformations of that vector space. Then a **Representation of** G **on** V is a homomorphism $\rho : G \to GL(V)$. If V is a vector space over a field F with $\dim_F V = n$ and we choose a basis for V, this is just a map from G to the set of invertable $n \times n$ matricies with coefficients in F that take the group operation to matrix multiplication. However, represnetation theory is more than just an occasionally useful way to write a group. It's also an incredibly rich area of study in its own right with application to topology, algebraic geometry, number theory, Lie theory, homological algebra, and mathematical physics.

We will start with representations of finite groups (see Artin Chapter 9) where we will cover invariant subspaces, irreducible representations, characters and character tables, the regular representation and Shur's lemma before moving to representations of infinite groups and then to **Lie groups** (groups that are also manifolds(!)) and **Lie algebras** (the Lie group's tangent space at the identity element).

Representation theory as a subject is comparatively young (only 100 years old or so) and we will potentially be able to discuss such new and exciting topics as the character tables in the Atlas of Finite Groups, the classification of simple complex Lie algebras (there are 5 exceptional groups!), Weyl groups, Killing forms, Dynkin diagrams, and the structure of local anomalies in gauge theories (high energy physics).

Here is my most important piece of advise about this course: This course is hard. The material is not just unfamiliar, but actively strange, so DO NOT FALL BEHIND!! As usual, this includes things like DO NOT MISS CLASS!! (If you must miss a class, get notes from one of your classmates *and read them* before the next class.) It also includes things like DO YOUR HOMEWORK!! It is not possible to actually learn this material without doing problems. You might be able to convince yourself you understand, but if you can't do problems, you aren't at the level of understanding required to pass the class. In fact, if the class seems too easy at any point, do extra problems!

GRADES: Your grade for this course will be determined by the big quizzes and your midterm (50% total), the final exam (30%), and by your written work (20%). This written work includes weekly quizzes and homework. Class participation is very important and will be counted with your written work.

EXAMS AND QUIZZES: There will be two big quizzes, a midterm and a final as well as short self-scheduled quizzes roughly weekly (usually on Fridays). Mark the dates on your calendar now. If you have an unavoidable conflict with one of the exam dates, let me know *as soon as possible* to arrange a makeup exam. Except in cases of sudden emergency, I will not arrange a makeup exam unless I know at least a week in advance.

Your weekly quizzes will be self scheduled. They involve getting the quiz out of an envelope in the computer lab next to my office, signing in, completing the quiz and putting it in a different envelope. I will go over the weekly quizzes in class the following Monday.

If you have any special needs, please see me in the first two weeks of the term.

HOMEWORK: there will also be homework due on Fridays and it MUST be typeset in TEX/LATEX. (Your homework can include hand drawn pictures or diagrams, but all words need to be type set.) Homework may be worked on in groups, but must be written up independently in your own words. Cooperation is encouraged but sharing TEX files is a serious HONOR CODE VIOLATION and I WILL prosecute if you do this. (Incidentally, shared syntax and typos are a dead give away for this violation.) Typically, I will assign homework weekly, and I will let you know at the time I assign written work when it is due.

As for the homework, I hope to be able to actually grade your homework (typesetting will help with this), however, the best way to get feedback on the homework is still to go over it with me, either in class (if it is a problem many people had trouble with) or in my office. Also, if you are concerned about getting your homeworks back, please scan them before you turn them in as a precaution because I can't guarantee any sort of timeliness. The quizzes will be your main source of weekly feedback.

If you do end up doing the bad-thing (cheating by looking up solutions on the internet), as a *minimum* to get anything educational out of the process (and to avoid an HONOR CODE VIOLATION) you must 1. reference your source, 2. change the wording of the solution (choose different English words) and 3. change the names of all of the variables. If I catch anyone copying from any source (including each other or me or your textbook) without doing those three things, I WILL bring you up on honors charges.

WEEKLY PROBLEMS TIME will be on Tuesday, hopefully during office hours. This is a time I will be available at a campus coffee facility for group and individual discussion of homework problems. I highly recommend you attend as many of these as possible, as well as work on the homework on your own *before* Tuesday.

TOPICS: Using Artin as a model, we will start in Chapter 5 with Group Actions, then reviewing Linear Algebra as necessary (Chapters 4, 7 and 8), move on to Representations of Finite Groups (Chapter 9). After this, we will talk about representations of infinite groups, Lie groups and Lie algebras, and whatever else we have time for (Chapter 8 and beyond)

ATTENDANCE and participation will be an important requirement of this course. If you must miss a class, be sure to get notes (and *read them before the next class*).

HONOR CODE: I take the honor code very seriously, and so should you. Any instances of suspected cheating or academic dishonesty will be referred to the JMU Honor Board for investigation. Please see the note at the end of the textbook section for specific requirements about academic honesty and homework.