Math 485 Homework 1

due Monday, January 23

1. Determine the group of automorphisms of the following groups:
   a) $\mathbb{Z}/2\mathbb{Z}$

   b) $\mathbb{Z}/6\mathbb{Z}$

   c) $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$

2. Let G be a group acting on a set X. Prove that $x \sim y$ if there exists $g \in G$ with $gx = y$ is an equivalence relation.

3. Let $\phi : G \to G'$ be a group homomorphism, and let X be a set on which G' acts. Show there exists an action of G on X.

4. Let $G = D_4$ the dihedral group of symmetries of the square.
   a) What is the stabilizer of a vertex? What is the stabalizer of an edge?

   b) G acts on the set of two elements consisting of the diagionals of the square. What is the stabalizer of a single diagional?

5. Let $G = GL_n(\mathbb{R})$ act on the set $\mathbb{R}^n$ by left multiplication.
   a) Describe the decomposition of X into orbits for this operation.

   b) What is the stabilizer of $e_1 = (1, 0, ..., 0)$, the first standard basis element?

6. Decompose the set $X = M_2(\mathbb{C})$ of all 2 by 2 square matricies with coefficients in $\mathbb{C}$ using the following actions of $GL_2(\mathbb{C})$ on X.
   a) Left multiplication.

   b) Conjugation