Logic and Proofs

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Implications

Definition

Let A and B be sentences.

- "If A, then B" is called an **implication**.
- A is called the **hypothesis** and B is called the **conclusion**.
- Also written "$A \Rightarrow B$" (read "A implies B").
- An implication is considered to be FALSE when A is true and B is false, and TRUE in every other instance.

Variations of Implications

Definition

- The **converse** of the implication "If A, then B" is the implication "If B, then A".
- The **inverse** of the implication "If A, then B" is the implication "If not A, then not B". (Also denoted $\neg A \to \neg B$.)
- The **contrapositive** of the implication "If A, then B" is the implication "If not B, then not A".

An implication and its contrapositive are *always* logically equivalent, meaning that if either of the statements is true the other is true as well.

Quantifiers

Definition

Suppose P is a property that depends on a value of x.

- "For all x we have property P" means that property P holds for every possible value of x. (Universal quantification)
- " There exists x such that we have property P" means that property P holds for at least one value of x. (Existential quantification)

Counterexamples

Definition

A **counterexample** is an example of a value that makes a statement false.

Theorem

- *Suppose P is a property that depends on a value of x. The statement "For all x, we have property P" is false if and only if we can find a counterexample for which P is false.*
- *Suppose A and B are statements that depend on a value of x. The statement "For all x, if A then B" is false if and only if we can find a counterexample for which A is true but is false.*