Graphing Polar Equations

Department of Mathematics and Statistics

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Using the θr-plane to Get Information About a Polar Graph

Recall that when we plot a point (r, θ) in polar coordinates, r represents the (signed) distance from the pole and θ represents the angular rotation (in radians) from the polar axis.

We use these properties to the graph a function of the form $r = f(\theta)$. However, to understand the graph, $r = f(\theta)$, using polar coordinates, it is often helpful to graph the equation first in the θr-plane.

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Symmetry in Polar Graphs

- A graph is symmetrical with respect to the x-axis if for every point (r, θ) on the graph, the point $(r, -\theta)$ is also on the graph.
- A graph is symmetrical with respect to the y-axis if for every point (r, θ) on the graph, the point $(-r, -\theta)$ is also on the graph.
- A graph is symmetrical with respect to the origin if for every point (r, θ) on the graph, the point $(-r, \theta)$ is also on the graph.

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Some Common Polar Graphs

- $r = a\sin\theta$ and $r = a\cos\theta$ – circles with radius $\frac{|a|}{2}$ containing the origin, tangent to the x-axis and y-axis, respectively.
- $r = a\cos k\theta$ and $r = a\sin k\theta$, with $|k|$ an integer greater than one – "roses" with $|k|$ petals if k is odd, and $2|k|$ petals if k is even.
- $r = a \pm b\cos\theta$ and $r = a \pm b\sin\theta$
  - cardioid when $\left|\frac{a}{b}\right| = 1$.
  - limaçon when $\left|\frac{a}{b}\right| \neq 1$.
  - with inner loop when $\left|\frac{a}{b}\right| < 1$.
- $r^2 = \pm a\sin 2\theta$ and $r^2 = \pm a\cos 2\theta$ – lemniscate.

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