Unit Tangent and Unit Normal Vectors

Department of Mathematics and Statistics

September 19, 2012

Unit Tangent Vectors

Unit Tangent Vectors

To understand the shape of a space curve we are often more interested in the direction of motion, that is, the direction of the tangent vector, rather than its magnitude. In this case we use the unit tangent vector:

Unit Tangent Vectors

To understand the shape of a space curve we are often more interested in the direction of motion, that is, the direction of the tangent vector, rather than its magnitude. In this case we use the unit tangent vector:

Definition

Let $\mathbf{r}(t)$ be a differentiable vector function on some interval $I \subseteq \mathbb{R}$ such that $\mathbf{r}'(t) \neq 0$ on I. The **unit tangent function** is defined to be

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}.$$

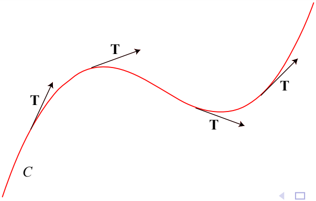
Unit Tangent Vectors

To understand the shape of a space curve we are often more interested in the direction of motion, that is, the direction of the tangent vector, rather than its magnitude. In this case we use the unit tangent vector:

Definition

Let $\mathbf{r}(t)$ be a differentiable vector function on some interval $I \subseteq \mathbb{R}$ such that $\mathbf{r}'(t) \neq 0$ on I. The **unit tangent function** is defined to be

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}.$$



The Principal Unit Normal Vector

The Principal Unit Normal Vector

Definition

Let $\mathbf{r}(t)$ be a differentiable vector function on some interval $I \subseteq \mathbb{R}$ such that the derivative of the unit tangent vector $\mathbf{T}'(t) \neq 0$ on I. The **principal unit normal vector** at $\mathbf{r}(t)$ is defined to be

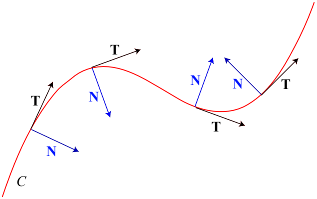
$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}.$$

The Principal Unit Normal Vector

Definition

Let $\mathbf{r}(t)$ be a differentiable vector function on some interval $I \subseteq \mathbb{R}$ such that the derivative of the unit tangent vector $\mathbf{T}'(t) \neq 0$ on I. The **principal unit normal vector** at $\mathbf{r}(t)$ is defined to be

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}.$$



The Osculating Plane and Binormal Vector

Definition

Let $\mathbf{r}(t) = \langle x(t), y(t), z(t)\rangle$ be a differentiable vector function on some interval $I \subseteq \mathbb{R}$ such that the derivative of the unit tangent vector $\mathbf{T}'(t_0) \neq 0$ where $t_0 \in I$. The **binormal vector B** at $\mathbf{r}(t_0)$ is defined to be

$$\mathbf{B}(t_0) = \mathbf{T}(t_0) \times \mathbf{N}(t_0),$$

and the **osculating plane** at $\mathbf{r}(t_0)$ is defined by

$$\mathbf{B}(t_0) \cdot \langle x - x(t_0), y - y(t_0), z - z(t_0)\rangle = 0.$$

The Osculating Plane and Binormal Vector

Definition

Let $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$ be a differentiable vector function on some interval $I \subseteq \mathbb{R}$ such that the derivative of the unit tangent vector $\mathbf{T}'(t_0) \neq 0$ where $t_0 \in I$. The **binormal vector B** at $\mathbf{r}(t_0)$ is defined to be

$$\mathbf{B}(t_0) = \mathbf{T}(t_0) \times \mathbf{N}(t_0),$$

and the **osculating plane** at $\mathbf{r}(t_0)$ is defined by

$$\mathbf{B}(t_0) \cdot \langle x - x(t_0), y - y(t_0), z - z(t_0) \rangle = 0.$$

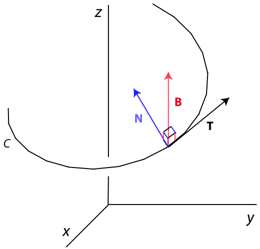
The Frenet or TNB Frame

The Frenet or TNB Frame

The **Frenet frame** is comprised of the three mutually perpendicular vectors $\mathbf{T}$, $\mathbf{N}$ and $\mathbf{B}$.

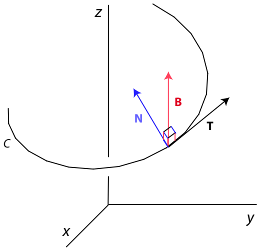
The Frenet or TNB Frame

The **Frenet frame** is comprised of the three mutually perpendicular vectors **T**, **N** and **B**.



The Frenet or TNB Frame

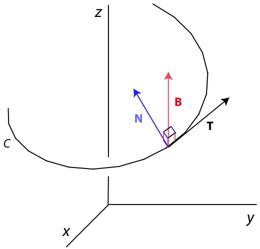
The **Frenet frame** is comprised of the three mutually perpendicular vectors **T**, **N** and **B**.



- This frame is different at each point on the space curve.

The Frenet or TNB Frame

The **Frenet frame** is comprised of the three mutually perpendicular vectors **T**, **N** and **B**.



- This frame is different at each point on the space curve.
- The osculating plane at a point P on the curve, is the unique plane containing P and the vectors **T** and **N**.