Discrete Mathematics
An Introduction to Proofs
Proof Techniques

Math 245
January 17, 2013

Proof Techniques

Proof Techniques

- Direct Proof

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- Direct Proof
- Indirect Proof

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- Direct Proof
- Indirect Proof
  - Proof by Contrapositive

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  - Proof by Contrapositive
  - Proof by Contradiction

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- Proof by Cases
- Existence Proof
- Proof by Induction

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  This can also help to identify the hypothesis and the conclusion.

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- That is, prove $\neg B \rightarrow \neg A$.

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- To summarize, to prove the implication $A \rightarrow B$ "by contradiction", we assume the hypothesis A and the negation of the conclusion $\neg B$ both hold.

  We show that this is a contradiction, so the original implication is a tautology.