Introduction to Vector Fields

Department of Mathematics and Statistics

November 7, 2012

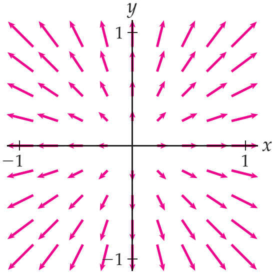
Vector Fields in $\mathbb{R}^2$

Definition

A **vector field** in $\mathbb{R}^2$ is a function $\mathbf{F}(x, y)$ with domain $D \subseteq \mathbb{R}^2$ and satisfying

$$\mathbf{F}(x, y) = \langle F_1(x, y), F_2(x, y) \rangle$$

for each point $(x, y) \in D$.



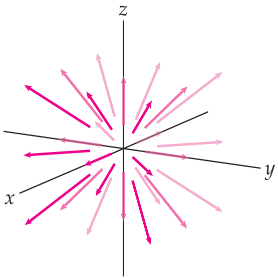
Vector Fields in $\mathbb{R}^3$

Definition

A **vector field** in $\mathbb{R}^3$ is a function $\mathbf{G}(x, y, z)$ with domain $D \subseteq \mathbb{R}^3$ and satisfying

$$\mathbf{G}(x, y, z) = \langle G_1(x, y, z), G_2(x, y, z), G_3(x, y, z)\rangle$$

for each point $(x, y, z) \in D$.



Conservative Vector Fields

Definition

A **conservative vector field F** is a vector field that can be written as a the gradient of some function f. That is,

$$\mathbf{F}(x, y) = \nabla f(x, y) = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j}$$

if $f(x, y)$ is a function of two variables, or

$$\mathbf{F}(x, y, z) = \nabla f(x, y, z) = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$$

if $f(x, y, z)$ is a function of three variables.

In either case, any function f whose gradient is equal to **F** is called a **potential function** for **F**.