Stokes' Theorem

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Theorem

Let $\mathcal{S}$ be a smooth or piecewise smooth oriented surface with a smooth or piecewise smooth boundary curve C. Suppose that $\mathcal{S}$ has an (oriented) unit normal vector $\mathbf{n}$ and that C has a parametrization that traverses C in the counterclockwise direction with respect to $\mathbf{n}$. If $\mathbf{F}(x,y,z) = F_1(x,y,z)\,\mathbf{i} + F_2(x,y,z)\,\mathbf{j} + F_3(x,y,z)\,\mathbf{k}$ is a vector field on an open region containing $\mathcal{S}$, then

$$\int_C \mathbf{F}(x,y,z) \cdot d\mathbf{r} = \iint_{\mathcal{S}} curl\, \mathbf{F}(x,y,z) \cdot \mathbf{n}\, dS.$$

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