

Initial Value Problems and Laplace Transform

Department of Mathematics and Statistics

September 4, 2012

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The Laplace Transform of the n th Derivative

Using mathematical induction, we can show:

$$\mathcal{L}\left(y^{(n)}\right) = -y^{(n-1)}(0) - s y^{(n-2)}(0) - \cdots - s^{n-1} y(0) + s^n \mathcal{L}(y).$$

We can apply this result to each side of a constant coefficient linear differential equation,

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 y' + a_0 y = g(t),$$

to turn the differential equation into an algebra problem:

$$a_n \mathcal{L}\left(y^{(n)}\right) + a_{n-1} \mathcal{L}\left(y^{(n-1)}\right) + \cdots + a_1 \mathcal{L}\left(y'\right) + a_0 \mathcal{L}(y) = \mathcal{L}(g(t)).$$

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