Convolution Integrals

Department of Mathematics and Statistics

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Introduction

Say we have a Laplace transform which is a product of two simpler functions and we know the inverse Laplace transform of these two simpler functions.

That is, we have

$$H(s) = F(s)G(s),$$

where we know $f(t) = \mathcal{L}^{-1}(F(s))$ and $g(t) = \mathcal{L}^{-1}(G(s))$.

How can we use this information to find $h(t) = \mathcal{L}^{-1}(H(s))$?

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The resulting function is called the **convolution product** of f and g and is denoted by $f * g$.

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