Euler Type Equations

Department of Mathematics and Statistics

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Definition

A second order differential equation of the form

$$(x - x_0)^2 y'' + \alpha(x - x_0)y' + \beta y = 0,$$

where α and β are constants, is called an **Euler type** equation or **equidimensional** equation.

Euler type differential equations of other orders may also be constructed, in the analogous way.

Note that x_0 is *not* an ordinary point of the differential equation.

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Using this power function in the differential equation, we obtain

$$r(r-1) + \alpha r + \beta = 0,$$

the **Euler indicial equation**.

The Euler Indicial Equation

The Euler indicial equation has roots:

$$r = \frac{1 - \alpha \pm \sqrt{(1-\alpha)^2 - 4\beta}}{2}.$$

There are three cases to consider, depending upon the discriminant, $(1-\alpha)^2 - 4\beta$, of the indicial equation:

1. $(1-\alpha)^2 - 4\beta > 0$, there are two distinct real roots.
2. $(1-\alpha)^2 - 4\beta = 0$, there is one repeated real root.
3. $(1-\alpha)^2 - 4\beta < 0$, there are two distinct complex roots.

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Solutions of Euler Type Equations

1. When there are two distinct real roots, the solutions are of the form:

$$y = x^{r_1} \text{ and } y = x^{r_2}.$$

2. When there is a single repeated real root, the solutions are of the form:

$$y = x^{r} \text{ and } y = x^{r}\ln x.$$

3. When there are two complex roots, the solutions are of the form:

$$y = x^{a}\cos(b\ln x) \text{ and } y = x^{a}\sin(b\ln x).$$

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