ANSWERS / HINTS

Exercises 1.1.

8. $y = 2 + 2(x + 3)$.

13(a). $y = 3 + \frac{7}{2}(x + 2)$.

13(b). $y = 3 - \frac{2}{7}(x + 2)$.

Exercises 1.2.

1(a). All x values.

1(d). $x \geq 0$.

1(f). All x values.

1(g). $x > 0$.

1(h). $x \neq 0$.

2. Domain: $x \geq 0$; range: $y = 0$ and $y > 1.25$.

5. $|x - 1| = x - 1$ if $x \geq 1$; $|x - 1| = -x + 1$ if $x < 1$.

7. $x \geq \frac{13}{3}$ or $x \leq -1$.

10(a). $x = \pm 3$.

10(d). $x = 0, 2, 5$.

11. $9^{1/2} = 3,\ 9^{3/2} = 27; 8^{1/3} = 2,\ 64^{-2/3} = \frac{1}{16}$.

12. $x = \frac{2}{3}$.

13. $x = -\frac{2}{9}$.

14. Let $y = 2^x$, then solve $y^2 - 5y + 4 = 0$ and get $y = 1$ and $y = 4$. From $y = 1$, we have $2^x = 1 = 2^0$, so that $x = 0$. From $y = 4$, we have $2^x = 4 = 2^2$, so that $x = 2$.

15(b). $\frac{f(w)-f(x)}{w-x} = \frac{2w^2-w-(2x^2-x)}{w-x} = \frac{2(w^2-x^2)-(w-x)}{w-x} = \frac{2(w-x)(w+x)-(w-x)}{w-x} = 2(w + x) - 1$.

15(c). $\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{x+h}-\sqrt{x}}{h} = \frac{[\sqrt{x+h}-\sqrt{x}][\sqrt{x+h}+\sqrt{x}]}{h[\sqrt{x+h}+\sqrt{x}]} = \frac{(x+h)-x}{h[\sqrt{x+h}+\sqrt{x}]} = \frac{h}{h[\sqrt{x+h}+\sqrt{x}]} = \frac{1}{\sqrt{x+h}+\sqrt{x}}$.

16. $f(g(x)) = (\sqrt{1-x})^5 = (1-x)^{5/2},\ g(f(x)) = \sqrt{1-x^5},\ f(f(x)) = (x^5)^5 = x^{25},\ g(g(x)) = \sqrt{1-\sqrt{1-x}}$.

22. Positive: $(-\infty,-2)$ and $(2,\infty)$. Negative: $(-2,2)$.

23(a). $(-\infty,1)$ and $(2,3)$.

23(b). $(-\infty,-4)$.

23(c). $(-7,-5)$ and $(6,9)$.

23(d). $(-\infty,-7)$, $(-5,6)$, and $(9,\infty)$.

Exercises 1.3.

9. $A(x)=100\left(1+\frac{0.03}{4}\right)^{4x}$.

13. Plug $t=4$.

Exercises 1.4.

1. $\log_5 125=3,\ \log_3\frac{1}{9}=-2,\ \log 1000=3$.

3. $x=[-5+\ln\frac{e^7}{3}]/4$ or $\frac{2-\ln 3}{4}$.

4. $x=\frac{5+\ln 3}{3}$.

5. $x=\frac{\ln 7-5\ln 3}{4\ln 3}$.

7. $x=\frac{5\ln 3}{7\ln 5-4\ln 3}$.

8. $\ln(\ln e^{e^e})=e$.

9. $\ln(AB)=1,\ \ln\frac{A}{B}=-5,\ \ln A^2=-4$.

11. $110\ln(3x+1)+220\ln(5x+2)-330\ln(7x+3)-440\ln(9x+4)$.

12. $\ln\frac{(2x)^x(7x+3)^{12}}{(3x-2)^{10}}$.

13. Find t from $36000e^{0.02t}=40000$.

14. Find t from $100(1+0.07)^t=200$.

15. We have $p(2)=20,600$ and $p(2)=20,000e^{2k}$, so we can solve k from $20600=20000e^{2k}$. Then plug $t=6$ to get the population in 2008.

Exercises 2.1.

1. $m=\frac{f(h)-f(0)}{h-0}=\frac{h^3-0}{h}=h^2$. For $h=1$, $m=h^2=1$. For $h=0.1$, $m=h^2=0.01$, and so on.

4. For $h=1$, average speed $=\frac{p(1+h)-p(1)}{h}=\frac{p(2)-p(1)}{1}=7$.

   For $h=0.1$, average speed $=\frac{p(1+h)-p(1)}{h}=\frac{p(1.1)-p(1)}{0.1}=3.31$, and so on.

5. See the examples in Section 2.3.

Exercises 2.2.

4(a)-(d). plug in.

4(e). $\frac{3}{5}$.

4(f). 4.

4(g). 3.

4(h). 4.

4(i). $\frac{1}{6}$.

4(j). plug in.

4(k). 0.

4(l). 1.

4(m). $\frac{5}{9}$.

4(n). 0.

2(o). $-\infty$; ∞.

2(p). DNE.

2(q). DNE.

5. 7.

6. $a = -2$.

7. $a = 1$, $b = 1$.

8. Do it case by case: $x = 0$, $x > 0$, and $x < 0$.

Exercises 2.3.

2(a). 0.

2(b). 2.

2(c). $2(x - 4)$.

2(d). $\frac{3}{2\sqrt{3x-1}}$.

2(e). $-\frac{2}{3x^2}$.

2(f). $-\frac{6}{(3x-1)^2}$.

2(g). $4 + \frac{3}{2x^2}$.

3(a). 0.

3(b). 2.

3(c). $2x - \frac{2}{3}x^{-1/3}$.

3(d). $\frac{1}{2}x^{-1/2} + 25x^4 - x^{-5/6}$.

3(e). $-\frac{2}{3x^2}$.

3(f). $4 + \frac{3}{2x^2}$.

10(a). 1234.

10(c). 80.

10(e). 108.

11. $\lim_{h\to 0} g(h) = f'(x_0),\ c = f'(x_0)$.

12. $c = f'(2)$.

Exercises 2.4.

1(a). $y = 3 + 2(x-1)$.

1(b). $y = 7 + 23(x-1)$.

1(c). $y = 3 + \frac{1}{6}(x-9)$.

1(d). $y = 2 + \frac{1}{12}(x-8)$.

2(a). $f(x) = x^{1/2}$; $a = 4$; tangent line: $y = 2 + \frac{1}{4}(x-4)$; 2.025.

2(d). $f(x) = x^{1/3}$, $a = 27$; tangent line: $y = 3 + \frac{1}{27}(x-27)$; 3.0037.

2(f). $f(x) = x^{-1/2}$; $a = 9$; tangent line: $y = \frac{1}{3} - \frac{1}{54}(x-9)$; 0.33241.

Exercises 3.1.

4. Example: $f(x) = x$, $g(x) = x$.

5. Example: $f(x) = x$, $g(x) = 1$.

9(e). $15x^4(x^{-2/5} - 3x) + (1 + 3x^5)(-\frac{2}{5}x^{-7/5} - 3)$.

9(k). $\frac{7x^6(4x^5+3x)-(x^7-5)(20x^4+3)}{(4x^5+3x)^2}$.

10(a). $f(x) = \frac{2-\sqrt{x}}{1+\sqrt{x}}$; $a = 9$; tangent line: $y = -\frac{1}{4} - \frac{1}{32}(x-9)$; -0.253125.

10(c). $f(x) = \frac{3+\sqrt[3]{x}}{1-\sqrt[3]{x}}$; $a = 8$.

Exercises 3.2.

1. Example: $f(x) = x^2$ (or $f(z) = z^2$), $g(x) = 2x$.

6(e). $2000(x - x^3)^{1999}(1 - 3x^2)$.

6(i). $34(2x+4)^{33}(2)(7x^2+8x-1)^{50} + (2x+4)^{34}50(7x^2+8x-1)^{49}(14x+8)$.

6(m). $F(x) = (3x + 2)^{50}(3x^6 - 7x)$, $G(x) = 8x^9 - 2$,

$$\begin{aligned} F'(x) &= 50(3x + 2)^{49}(3)(3x^6 - 7x) + (3x + 2)^{50}(18x^5 - 7), \\ G'(x) &= 72x^8. \end{aligned}$$

Then put them into the quotient rule.

6(r). $g(x) = (7x^2 + 1)^{40}(x^3 - 5) - \frac{x^{-3/2}}{7x^3+6x-1}$,

$$g'(x) = 40(7x^2 + 1)^{39}(14x)(x^3 - 5) + (7x^2 + 1)^{40}(3x^2) - \frac{-\frac{3}{2}x^{-5/2}(7x^3 + 6x - 1) - x^{-3/2}(21x^2 + 6)}{(7x^3 + 6x - 1)^2}.$$

Then put it into the general power rule.

7(a). $y = 1 - 332x$.

7(c). $y = -1 - 108x$.

8(a). $f(x) = (2 - \sqrt{x})^3(1 + \sqrt{x})^2$; $a = 9$; tangent line: $y = -16 - \frac{28}{3}(x - 9)$; -16.93.

Exercises 3.3.

1(a). $y'(x) = \frac{-y-3x^2}{3y^2+x}$.

1(c). $y'(x) = \frac{-3(y+x+6)^2-4xy^2-4x^3}{3(y+x+6)^2+4x^2y+1}$.

2. Tangent line: $y = 1 - \frac{19}{18}(x - 2)$; $y(2.01) \approx 1 - \frac{19}{18}(2.01 - 2) \approx 0.9894$.

5. Note that x and u are both functions of t, so we can take a derivative in t to the equation. (a). $x'(t) = \frac{97}{50} \approx 2$ *units/month*. (b). $u'(t) = \frac{150}{48.5} \approx 3$ *\$/month*.

9. Let $x = x(t)$ be the horizontal distance and $y = y(t)$ be the vertical distance, then we have a right triangle. Thus, $x^2 + y^2 = 30^2$, and we know that $x'(t) = 2$ ft/sec, and we want to find $y'(t)$ when $x = 20$. From $x^2+y^2 = 30^2$, we take a derivative in t and obtain $2xx'+2yy' = 0$. Thus, $y' = -\frac{xx'}{y} = -\frac{(20)(2)}{\sqrt{30^2-20^2}} = -\frac{4}{\sqrt{5}}$ ft/sec when $x = 20$.

10. Let l be the length of a side. Using right triangles, the area of the equilateral triangle is given by $A = \frac{\sqrt{3}l^2}{4}$, where A and l are both functions of t. Then, take a derivative in t and plug $l = 10$.

12. Let r be the radius of the circle and let x be the side of the square. Then, using right triangles, we get $x^2 + x^2 = (2r)^2$ or $x^2 = 2r^2$, where x and r are both functions of t. Then take a derivative in t and plug $r = 20$.

Exercises 4.1.

4(a). $4e^{4x} + 3x^2 - e^x$.

4(h). $F(x) = e^{5x^7+8x}$, $G(x) = \sqrt{x^3+2} + x^{-3/4}$,

$$\begin{aligned} F'(x) &= e^{5x^7+8x}(35x^6+8), \\ G'(x) &= \frac{1}{2}(x^3+2)^{-1/2}(3x^2) - \frac{3}{4}x^{-7/4}. \end{aligned}$$

Then put them into the quotient rule.

4(k). $40(x^{-5/3} + e^{7x^3+5x})^{39}[-\frac{5}{3}x^{-8/3} + e^{7x^3+5x}(21x^2+5)]$.

Exercises 4.2.

1. $P(t) = 76e^{0.03t}$.

2. $P(t) = 2e^{2t/5}$.

4. $P(t) = 750e^{0.05t}$; $P(12) = 750e^{0.05 \cdot 12}$; find t such that $750e^{0.05t} = 1500$.

5. $P(t) = 55300e^{kt}$ and then solve k from $55300e^{k \cdot 19} = 75400$; $P(29) = 55300e^{k \cdot 29}$; find t such that $55300e^{kt} = 99000$.

7. Find t such that $P(0)e^{-0.02t} = \frac{1}{3}P(0)$.

8. Find k from $P(0)e^{k \cdot 250} = \frac{1}{2}P(0)$, then find t such that $P(0)e^{kt} = \frac{1}{4}P(0)$.

Exercises 4.3.

1(a). $3x^2 \ln x + x^3 \frac{1}{x} = 3x^2 \ln x + x^2$.

1(b). $3x^2 \log_5 x + x^3 \frac{1}{x \ln 5} = 3x^2 \log_5 x + \frac{x^2}{\ln 5}$.

1(e). $F(x) = x - \ln 3x$, $G(x) = \ln x - x$,

$$\begin{aligned} F'(x) &= 1 - \frac{1}{x}, \\ G'(x) &= \frac{1}{x} - 1. \end{aligned}$$

Then put them into the quotient rule.

1(i). $F(x) = e^x \ln 3x$, $G(x) = x^2 - 2x$,

$$\begin{aligned} F'(x) &= e^x \ln 3x + e^x \frac{1}{x}, \\ G'(x) &= 2x - 2. \end{aligned}$$

Then put them into the quotient rule.

1(l). $\frac{7}{\ln 10} \frac{3+e^{x^2}2x}{3x+e^{x^2}-2}$.

1(p). $\frac{6}{x} - 7\frac{3+e^{x^2}2x}{3x+e^{x^2}}$.

2(c). $(x^3+x)^x[\ln(x^3+x) + x\frac{3x^2+1}{x^3+x}]$.

2(g). $4^{4^x}4^x(\ln 4)^2$.

2(l). $x^{x^x}[x^x(\ln x + 1)\ln x + x^{x-1}]$.

2(n). $\frac{(x^2+1)^{10}(x^2+2)^{20}(x^2+3)^{30}}{(x^2+4)^{40}(x^2+5)^{50}}\left\{10\frac{2x}{x^2+1} + 20\frac{2x}{x^2+2} + 30\frac{2x}{x^2+3} - 40\frac{2x}{x^2+4} - 50\frac{2x}{x^2+5}\right\}$.

Exercises 5.1.

3(a). Decreasing: $(-\infty, -1)$. Increasing: $(-1, \infty)$. Global minimum at $x = -1$.

3(c). Increasing: $(-\infty, 1)$, $(2, \infty)$. Decreasing: $(1, 2)$. Local maximum at $x = 1$. Local minimum at $x = 2$.

3(e). Increasing: $(-\infty, \infty)$. No maximum/minimum values.

3(k). Decreasing: $(-\infty, 0)$. Increasing: $(0, \infty)$. Global minimum at $x = 0$.

3(n). Increasing: $(-\infty, \infty)$. No maximum/minimum values.

3(q). Increasing: $(0, e)$. Decreasing: (e, ∞). Global maximum at $x = e$.

Exercises 5.2.

2(a). Concave up: $(-\infty, \infty)$. Global minimum at $x = -1$.

2(c). Concave down: $(-\infty, \frac{3}{2})$. Concave up: $(\frac{3}{2}, \infty)$. Inflection point at $x = \frac{3}{2}$. Local maximum at $x = 1$. Local minimum at $x = 2$.

2(e). Concave down: $(-\infty, 3)$. Concave up: $(3, \infty)$. Inflection point at $x = 3$. No maximum/minimum values.

2(k). Concave up: $(-\infty, \infty)$. Global minimum at $x = 0$.

2(n). Concave up: $(-\infty, -2)$. Concave down: $(-2, \infty)$. No maximum/minimum values.

2(q). Concave down: $(0, e^{3/2})$. Concave up: $(e^{3/2}, \infty)$. Global maximum at $x = e$.

Exercises 5.3.

The functions here are the same as those in exercises 5.1 and 5.2. So the solutions in exercises 5.1 and 5.2 can be used to sketch the functions here.

Exercises 5.4.

1. $x = 16$.

2. $x = 1125$.

5. \$6 per book.

7. $x = 32$, $w = 32$.

8. $x = 8$, $w = 8$.

9. $x = 15$, $w = 30$.

10. $x \approx 25.8$, $w \approx 19.4$.

11. $r = \frac{216}{6\pi}$, $h = 36$.

12. $r = \left(\frac{V}{2\pi}\right)^{1/3}$, $h = 2r = 2\left(\frac{V}{2\pi}\right)^{1/3}$.

Exercises 6.1.

1. $\int_0^2 f(x)dx \approx (\frac{1}{3})^2\frac{2}{3} + 1^2\frac{2}{3} + (\frac{5}{3})^2\frac{2}{3}$.

2(a). $\sum_{i=0}^{3} 2^i = 1 + 2 + 4 + 8$.

2(c). $\sum_{i=0}^{3}(-1)^{i+2}2^{2i} = 1 - 4 + 16 - 64$.

3(a). $1 + 3 + 5 + 7 + \cdots + 611 = \sum_{k=0}^{305}(2k + 1)$.

7. $\int_2^4(6x + 5)dx = 46,\ F(x) = 3x^2 + 5x$.

Exercises 6.2.

1(b). $\int(x^{-2/3} - x^2 + 8)dx = 3x^{1/3} - \frac{x^3}{3} + 8x + C$.

1(c). $\int(x^{1/2} - \frac{4}{5x})dx = \frac{2}{3}x^{3/2} - \frac{4}{5}\ln|x| + C$.

1(d). $\int(e^4 - e^{-4x} + \frac{3}{x^3})dx = e^4x + \frac{e^{-4x}}{4} - \frac{3}{2}x^{-2} + C$.

2. The area $= \int_1^2(\sqrt{x} + \frac{8}{3x} + 2 + e^{3x})dx = ...$

6(a). $\int_0^1(x^{2/5} + 8x^2 - 8)\,dx = (\frac{5}{7}x^{7/5} + \frac{8}{3}x^3 - 8x)\Big|_0^1 = \frac{5}{7} + \frac{8}{3} - 8$.

6(c). $\int_1^e (x^{1/2} - \frac{4}{5x})\,dx = (\frac{2}{3}x^{3/2} - \frac{4}{5}\ln|x|)\Big|_1^e = \frac{2}{3}e^{3/2} - \frac{4}{5} - \frac{2}{3}$.

6(g). $\int_1^2 (5x^4 + 6x^2 - 6x^{-5/6} + \frac{9}{x})\,dx = ...$

7. $p(t) = p(0) + \int_0^t \frac{2}{5+3s} ds = 0 + \frac{2}{3}\ln(5+3s)\Big|_0^t = \frac{2}{3}\ln(5+3t) - \frac{2}{3}\ln 5$.

9. $C(x) = C(0) + \int_0^x (s^2 - 22s + 160) ds = ...$

15. $f(x) = \int(e^{2x} - x^{-2/3} + 3)dx = \frac{e^{2x}}{2} - 3x^{1/3} + 3x + C$ and $f(0) = -4$. Thus, $\frac{1}{2} + C = -4$, and $c = -4 - \frac{1}{2} = -\frac{9}{2}$. Therefore, $f(x) = \frac{e^{2x}}{2} - 3x^{1/3} + 3x - \frac{9}{2}$.