The Monty Hall Problem, Reconsidered

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In its classical form, the Monty Hall Problem (MHP) is the following:

Version 1 (Classic Monty)**.** *You are a player on a game show and are shown three identical doors. Behind one is a car, behind the other two are goats. Monty Hall, the host of the show, asks you to choose one of the doors. You do so, but you do not open your chosen door. Monty, who knows where the car is, now opens one of the doors. He chooses his door in accordance with the following rules:*

1. *Monty always opens a door that conceals a goat.*
2. *Monty never opens the door you initially chose.*
3. *If Monty can open more than one door without violating rules one and two, then he chooses his door randomly.*

After Monty opens his door, he gives you the options of sticking with your original choice, or switching to the other unopened door. What should you do to maximize your chances of winning the car?

In the entire annals of mathematics you would be hard-pressed to find a problem that arouses the passions like the MHP. It has a history going back at least to 1959, when Martin Gardner introduced a version of it in *Scientific American* [4, 5]. When statistician Fred Moseteller included it in his 1965 anthology of probability problems [9], he remarked that it attracted far more mail than any other problem. Writing in his 1968 book *Mathematical Ideas in Biology* [16], biologist John Maynard Smith wrote, "This should be called the Serbelloni problem since it nearly wrecked a conference on theoretical biology at the villa Serbelloni in the summer of 1966." In its modern game show format the problem made its first appearance in a 1975 issue of the academic journal *The American Statistician* [14]. Mathematician Steve Selvin presented it as an interesting classroom exercise on conditional probability. Though he presented the correct solution, (that there is a big advantage to be gained from switching), he found himself strongly challenged by subsequent letters to the editor [15].

The problem really came into its own when *Parade* magazine columnist Marilyn vos Savant responded to a reader's question regarding it. There followed several rounds of angry correspondence, in which readers challenged vos Savant's solution. The challengers later had to eat crow when it was shown by a Monte Carlo simulation that vos Savant was correct, but not before the fracas reached the front page of the *New York Times* [18]. The whole story is recounted in [13].

In the end, the situation has been best summed up by cognitive scientist Massimo Palmatelli-Palmarini who wrote that, "...no other statistical puzzle comes so close to fooling all the people all the time... The phenomenon is particularly interesting precisely because of its specificity, its reproducibility, and its immunity to higher education." [10]

Why All the Confusion? The trouble, you see, is that most people argue like this: "Once Monty opens his door only two doors remain in play. Since these doors are equally likely to be correct, it does not matter whether you switch or stick." We will refer to this as the fifty-fifty argument.

This intuition is supported by a well-known human proclivity. A negative consequence incurred by inaction hurts less than the same negative consequence incurred through some definite action. In the context of the MHP, people feel worse when they switch and lose than they do after losing by sticking passively with their initial choice.

There is a large literature in the psychology and cognitive science journals documenting and explaining the difficulty people have with the MHP. Burns and Wieth [3] summarized the findings of numerous such studies by writing,

> These previous articles reported 13 studies using standard versions of the MHD, and switch rates ranged from 9% to 23% with a mean of 14.5%. This consistency is remarkable given that these studies range across large differences in the wording of the problem, different methods of presentation, and different languages and cultures.

(Note that MHD stands for "Monty Hall Dilemma.")

Gilovich, Medvec, and Chen [6] studied people's reactions to losing by switching versus their reactions to losing by sticking. They used boxes instead of doors, and crafted an experimental situation in which players would lose regardless of their decision to switch or stick. Their findings?

> Because action tends to depart from the norm more than inaction, the individual is likely to feel more personally responsible for an unfortunate action. Thus, subjects who switched boxes in our experiment were more likely to experience a sense of "I brought this on myself," or "This need not have happened," than subjects who decided to keep their initial box.

It would seem the defenders of sticking can point both to a plausible mathematical argument and to certain fine points of human psychology. How can the forces for switching fight back?

Focus on Monty, Not the Doors There are a variety of elementary methods for solving the MHP. Working out the tree diagram for the problem, as shown in Figure 1, establishes that switching wins with probability $\frac{2}{3}$, while sticking wins with probability $\frac{1}{3}$. Consequently, we double our chances of winning by switching.

Monte Carlo simulations are also effective for establishing the correct answer. The Monty Hall scenario is readily simulated on a computer. The large advantage to be gained from switching quickly becomes apparent by playing the game multiple times.

Such methods, however, do little to clarify why the fifty-fifty argument is incorrect. Practical results obtained from a simulation can show you that *something* is wrong with your intuition, but they will not make the correct answer seem natural. The trouble lies in the difficulty people have in recognizing what is and is not important in reasoning about conditional probability.

The mantra in the title of this section goes a long way towards pointing people in the right direction. When Monty opens door X, there is a tendency to think, "I have learned that door X conceals a goat, but I have learned nothing of relevance about the other two doors." This is what we mean by "focusing on the doors." The proper approach involves focusing on Monty, specifically on the precise manner in which he chooses his door to open. We should think, "Monty, who makes his decisions according to strict rules, chose to open door X. Why this door as opposed to one of the others?"

Let us assume the player initially chose door one and Monty then opened door two. According to the rules, we can be certain that one of the following two scenarios has played out:

1. The car is behind door one. Monty chose door two at random from among doors two and three.

2. The car is behind door three. Since the player initially chose door one, Monty was now forced to open door two.

The second of these scenarios is more likely than the first. Since the car is behind the first door one-third of the time, and since Monty then opens door two in one-half of those cases, we see that scenario one occurs one-sixth of the time. Scenario two, on the other hand, happens whenever the car is behind door three (and the player has chosen door one). That happens one-third of the time. Scenario two is twice as likely as scenario one.

Thus, we should think, "I have just witnessed an event that is twice as likely to occur when the car is behind door three than it is when the car is behind door one. Consequently, the car is more likely to be behind door three, and I am more likely to win the car by switching."

An Exotic Selection Procedure The general principle here is that anything affecting Monty's decision-making process is relevant to updating our probabilities after Monty opens his door. To further illuminate this point, let us consider an altered version of the problem:

Version 2 (High-Numbered Monty)**.** *As before, we have three identical doors concealing one car and two goats. The player chooses a door which remains unopened. Monty now opens a door he knows to conceal a goat. This time, however, we stipulate that Monty always opens the highest-numbered door available to him (keeping in mind that Monty will never open the door the player chose.) Will the player gain any advantage by switching doors?*

For reasons of concreteness, we will assume once more that the player initially chooses door one.

Any time door one conceals a goat, Monty has no choice regarding which door to open. He can not open door one (since the player chose that door), and he can not open the door that conceals the car. This leaves only one door available to him.

The interesting cases occur when door one conceals the car. Unlike Classic Monty, who now chooses randomly, High-Numbered Monty will always open door three when he can. It follows that if we see him open door two instead we know for certain that the car is behind door three.

And if High-Numbered Monty opens door three? Since Monty is certain to open door three whenever the car is behind door one or door two, we now have no basis for deciding between them. It really is a fifty-fifty decision in this case.

Take this as a cautionary tale. Whether we are playing Classic Monty or High-Numbered Monty, it is certain that Monty will open a goat-concealing door. In the former case the probability that our initial choice concealed the car did not change while in the latter case it did. This shows that any proposed solution to the MHP failing to pay close attention to Monty's selection procedure is incomplete.

Monty Meets Bayes The main point thus far is that the probability that door X conceals the car given that Monty has shown us the goat behind door Y depends on a detailed consideration of Monty's selection procedure. More precisely, it depends on the probability that Monty will open door Y under the assumption that door X conceals the car. The precise manner in which these probabilities are related is given by Bayes' theorem.

We denote by C_i the event that the car is behind door i, and by M_j the event that Monty opens door j to reveal a goat. Also assume the player initially chooses door one, and Monty then opens door two. Bayes' theorem tells us that

$$P(C_1|M_2) = \frac{P(C_1)P(M_2|C_1)}{P(M_2)}.$$

Expanding the bottom of this fraction via the law of total probability leads to

$$P(C_1|M_2) = \frac{P(C_1)P(M_2|C_1)}{P(C_1)P(M_2|C_1) + P(C_2)P(M_2|C_2) + P(C_3)P(M_2|C_3)}.$$

In both of our versions of the MHP we have $P(M_2|C_2) = 0$, since it is given that Monty will never open the door concealing the car. Also, since we are given the doors are identical, we have

$$P(C_1) = P(C_2) = P(C_3) = \frac{1}{3}.$$

Making these substitutions leads to

$$P(C_1|M_2) = \frac{P(M_2|C_1)}{P(M_2|C_1) + P(M_2|C_3)}.$$

In both versions of the game we have $P(M_2|C_3) = 1$. That is, when the player chooses door one and the car is behind door three, Monty is certain to open door two.

In Classic Monty we have $P(M_2|C_1) = \frac{1}{2}$, since Monty chooses at random when the car is behind the door initially chosen by the player. In High-Numbered Monty we have $P(M_2|C_1) = 0$, since Monty is required by his rules to open door three. Plugging everything into Bayes' Theorem shows that for Classic Monty we now have

$$P(C_1|M_2) = \frac{\frac{1}{2}}{\frac{1}{2} + 1} = \frac{1}{3},$$

while for High-Numbered Monty we have

$$P(C_1|M_2) = \frac{\frac{1}{2}}{0 + 1} = \frac{1}{2}.$$

These are precisely the answers we obtained in the previous section.

Let us go one more round:

Version 3 (Random Monty)**.** *As always, assume that the player has initially chosen door one and Monty subsequently opened door two to reveal a goat. This time, however, you know that Monty chose his door randomly, subject only to the restriction that he not open the door the player chose. Should we switch?*

The novelty here lies in the nonzero probability of Monty opening the door concealing the car. Intuitively we would reason as follows: Since Monty opened door two after I selected door one, since door two concealed a goat, and since I know Monty chose randomly between doors two and three, I conclude that one of two scenarios has played out:

1. The car is behind door one, Monty chose door two randomly.

2. The car is behind door three, Monty chose door two randomly.

Since the car is equally likely to be behind doors one and three, these scenarios are equally likely to occur. The conclusion is that the remaining doors are equiprobable, and therefore there is no advantage to switching.

Our intuition is confirmed via Bayes' Theorem. We know that Monty will not open door one, and we know that door two conceals a goat. We now have

$$\begin{aligned} P(C_1) = P(C_2) = P(C_3) &= \frac{1}{3}, \\ P(M_2|C_1) = P(M_2|C_3) &= \frac{1}{2} \\ P(M_2|C_2) &= 0. \end{aligned}$$

Bayes' Theorem now says

$$P(C_1|M_2) = \frac{\frac{1}{3}\left(\frac{1}{2}\right)}{\frac{1}{3}\left(\frac{1}{2}\right) + \frac{1}{3}(0) + \frac{1}{3}\left(\frac{1}{2}\right)} = \frac{1}{2}.$$

The tree diagram in Figure 2 might be helpful for visualizing the situation.

Two-Player Monty Three-door versions of the MHP can become remarkably complex. The following version comes from a paper by philosopher Peter Baumann [1].

Version 4 (Two-Player Monty)**.** *We begin with three identical doors concealing two goats and one car. There are two players in the game. Each player chooses one of the doors but does not open it. Each player knows there is another person in the game, but neither knows which door the other player selected. Monty now opens a door according to the following procedure.*

1. *If both players selected the same door, then everything proceeds as in the classical game. Monty opens a goat-concealing door, choosing randomly if he has a choice.*

2. *If the players selected different doors, then Monty opens the one remaining door, regardless of what is behind it.*

You can assume that both players select their initial doors randomly. If you are one of the players and you have just seen Monty open a goat-concealing door, should you switch?

This will be a fine test of our new-found intuition. How do things look from the perspective of Player A? For concreteness, suppose that Player A initially chose door one, and Monty has now opened the goat-concealing door two. We reason that one of three scenarios has played out:

1. Player B chose door one, door one conceals the car, Monty chose door two randomly.

2. Player B chose door one, door three conceals the car, Monty was forced to open door two.

3. Player B chose door three, Monty was forced to open door two.

Prior to Monty's actions we would have considered it equally likely that Player B chose door one, door two or door three. After Monty's actions, we can dismiss the possibility that Player B chose door two. I claim that we should now think it is more likely that Player B chose door three than that he chose door one. We consider several cases.

- If player B chose door three, then Monty is forced to open door two. It conceals a goat with probability $\frac{2}{3}$.

- If Player B chose door one then there are two further cases to consider.

  – The car is behind door one with probability $\frac{1}{3}$. In this case, Monty opened door two randomly, which happens with probability $\frac{1}{2}$. It follows that scenario one above happens with probability $\frac{1}{6}$.

- The car is behind door three with probability $\frac{1}{3}$. In this case, Monty is forced to open the goat-concealing door two. It follows that the probability that Player B chose door one is $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$.

The event in which Monty opens the goat-concealing door two after Player A chooses door one is more likely to occur when Player B has chosen door one than when he has chosen door three. Specifically, it is $\frac{4}{3}$ more likely that Player B has chosen door three.

It is a consequence of Bayes' theorem that the probabilities we now assign to "Player B chose door three," and "Player B chose door one," must preserve this $4 : 3$ ratio. (A proof of this assertion can be found in the paper by Rosenthal [12].) Consequently, we assign probabilities of $\frac{4}{7}$ and $\frac{3}{7}$ respectively.

To continue the analysis, note that from Player A's perspective there are now four possibilities. Player B could have chosen door one or door three, and the car could be behind either of those doors. Let us denote these possibilities via ordered pairs of the form

$$(\text{Player } B\text{'s Door, Location of the Car}).$$

Thus, the four remaining possibilities are

$$(3,1), \quad (3,3), \quad (1,1), \quad (1,3).$$

Consider the first two pairs. If Player B chose door three, then Monty was forced to open door two. Consequently, we learn nothing regarding the probability of doors one and three. Since these two scenarios collectively have a probability of $\frac{4}{7}$, and since they are equally likely, we now assign the following probabilities:

$$P(3,1) = P(3,3) = \frac{2}{7}.$$

The remaining two pairs, however, are not equiprobable. Suppose that Player B chose door one, just as Player A did. If the car is behind door one then Monty chose door two randomly, which happens with probability $\frac{1}{2}$. If the car is behind door three then Monty was forced to choose door two. It follows that it is twice as likely that the car is behind door three than that it is behind door two. Since these scenarios have a collective probability of $\frac{3}{7}$, we assign the following probabilities:

$$P(1,1) = \frac{1}{7} \text{ and } P(1,3) = \frac{2}{7}.$$

Of the four scenarios, the two in which Player A wins by switching are $(1,3)$ and $(3,3)$. Since both have probability $\frac{2}{7}$, this gives a total probability of winning by switching of $\frac{4}{7}$. That is our solution.

The really amusing part is that both players will go through this analysis, and both will decide to switch doors. In those scenarios in which the players chose different doors, this implies that someone is definitely making the wrong decision. Such are the cruelties of probability.

Two-player Monty has also been discussed by Levy [7], Baumann [1], Rosenhouse [11] and Sprenger [17].

Many Doors Ready for the final exam?

Version 5 (Progressive Monty)**.** *This time there are n identical doors, concealing one car and $n-1$ goats. The player chooses a door, but does not open it. Monty now opens a goat-concealing door, choosing randomly from among his options. The player is now given the choice of sticking or switching. The player makes his choice, but again does not open his chosen door. Monty opens another goat-concealing door. The player is again given the opportunity to stick or switch. This continues until Monty has opened $n-2$ doors. The player makes his final selection, and wins whatever is behind his door. What strategy will maximize his chances of winning the car?*

In discussing this version it will be convenient to refer simply to the probability of a specific door. By this we mean the probability that the given door conceals the car.

To help us get our bearings, let us try a case study. Suppose we begin with five doors. At any stage of the game we shall represent the probabilities of the doors via an ordered 5-tuple, which we will refer to as the probability vector. As the game begins we have probability vector

$$\left(\frac{1}{5},\ \frac{1}{5},\ \frac{1}{5},\ \frac{1}{5},\ \frac{1}{5}\right).$$

As always, let us assume the player chooses door one and Monty now opens door two. Our past experience suggests that the probability of our door does not change, and this is confirmed by Bayes' theorem. In the following calculation, the notation $\overline{C_i}$ denotes the event where the car is not behind door i.

We now compute:

$$\begin{aligned} P(C_1|M_2) &= \frac{P(C_1)P(M_2|C_1)}{P(C_1)P(M_2|C_1) + P(\overline{C_1} \text{ and } \overline{C_2})P(M_2|\overline{C_1} \text{ and } \overline{C_2})} \\ &= \frac{\frac{1}{5}\left(\frac{1}{4}\right)}{\frac{1}{5}\left(\frac{1}{4}\right) + \frac{3}{5}\left(\frac{1}{3}\right)} = \frac{1}{5}. \end{aligned}$$

Since the other doors are identical and since their probabilities must sum to $\frac{4}{5}$, we now have probability vector

$$\left(\frac{1}{5},\ 0,\ \frac{4}{15},\ \frac{4}{15},\ \frac{4}{15}\right).$$

What if we now switch to door three and then see Monty open door five? We now have the values:

$$\begin{aligned} P(C_3) = P(C_4) = P(C_5) &= \frac{4}{15} \\ P(M_5|C_1) = P(M_5|C_4) &= \frac{1}{2} \\ P(C_1) = \frac{1}{5} \text{ and } P(M_5|C_3) &= \frac{1}{3}. \end{aligned}$$

If we use the law of total probability to write

$$P(M_5) = P(C_1)P(M_5|C_1) + P(C_3)P(M_5|C_3) + P(C_4)P(M_5|C_4) = \frac{29}{90},$$

and plug the results into Bayes' Theorem, the result is the probability vector

$$\left(\frac{9}{29},\ 0,\ \frac{8}{29},\ \frac{12}{29},\ 0\right).$$

It seems that the probabilities of all the remaining doors went up.

What if Monty had opened door one after we switched to door three? The reader can supply the details that lead to the vector

$$\left(0,\ 0,\ \frac{1}{4},\ \frac{3}{8},\ \frac{3}{8}\right).$$

Notice that the probability of door three went down, from $\frac{4}{15}$ to $\frac{1}{4}$. Our chosen door actually seems less likely as the result of Monty's actions. A surprising result!

Things get messy indeed in this version. Plainly we need some guidelines to aid our intuition.

The first principle is simple. Any time Monty chooses not to open a door different from your present choice, the probability of that door goes up. In our case study, Monty opened door two after we chose door one. The event, "Monty does not open door three," is more likely to happen when the car is behind door three than when it is elsewhere. Consequently, we will revise upward our probability of door three.

The second principle is that if the doors different from your present choice are equiprobable, then the probability of your choice does not change when Monty opens a door. In our case study, after Monty opened door two, we reason that the event, "Monty does not open door one," has probability one regardless of the location of the car. Consequently, we learn nothing from the occurrence of that event. The calculation in our case study confirms this intuition.

Why, though, does it matter that the other doors are equiprobable? The answer is that Monty's failure to open a door is not the only source of information to which we have access. The probability of the event, "Monty opens door X," depends in part on the probability of the event, "Door X

conceals the car.” Specifically, the more likely a door is to conceal the car, the less likely Monty is to open that door. Once more returning to our case study, we switched to door three at a moment when doors three through five were equiprobable and collectively very likely to conceal the car. By opening door five, Monty eliminated one element of this collection. This revelation does nothing to shake our confidence that the car is more likely to be found among doors three through five than it is to be found among any collection of three doors that includes door one. Consequently, we will revise upward the probability of our chosen door.

On the other hand, the probability of door three went down when Monty opened door one. ?????????????????????

This observation leads to our final clue. If we select a door at a moment when precisely k doors remain, the probability of that door can never be smaller than $\frac{1}{k}$. Even if we have been careless in extracting the maximum amount of information from Monty’s actions, we still know the door was chosen from among k possibilities.

As a test of our principles, let us go another round with our case study. We left off with the player having chosen door three and with probability vector,

$$\left(0,\ 0,\ \frac{1}{4},\ \frac{3}{8},\ \frac{3}{8}\right).$$

Imagine that we now switch to door four.

If Monty now opens door three then only doors four and five remain in play. We would reason that these two doors were equiprobable at the moment we switched to door four, but that door four was selected from among three possibilities. We are, in effect, playing Classic Monty, and we would expect our updated probability vector to be

$$\left(0,\ 0,\ 0,\ \frac{1}{3},\ \frac{2}{3}\right).$$

The calculation is:

$$P(C_4|M_3) = \frac{P(C_4)P(M_3|C_4)}{P(C_4)P(M_3|C_4) + P(C_5)P(M_3|C_5)}$$
$$= \frac{\frac{3}{8}\left(\frac{1}{2}\right)}{\left(\frac{3}{8}\right)\left(\frac{1}{2}\right) + \left(\frac{3}{8}\right)(1)} = \frac{1}{3}.$$

And if Monty opens door five instead? Our intuition tells us that both doors should see their probabilities go up: door three, because it might have been opened but was not; door five, because it was part of an equiprobable ensemble which has decreased in size. Bayes' Theorem confirms our intuitions. We compute

$$P(C_3|M_5) = \frac{P(C_3)P(M_5|C_3)}{P(C_3)P(M_5|C_3) + P(C_4)P(M_5|C_4)} = \frac{\frac{1}{4}(1)}{\frac{1}{4}(1) + \frac{3}{8}\left(\frac{1}{2}\right)} = \frac{4}{7},$$

and obtain probability vector

$$\left(0,\ 0,\ \frac{4}{7},\ \frac{3}{7},\ 0\right).$$

Remarkably, our arguments to this point are already enough to justify the correct solution. Consider the strategy in which we switch at the last minute (SLM). That is, we will stick with our initial choice until only two doors remain, and then we will switch. Our initial choice has probability $\frac{1}{n}$. Since the other doors are equiprobable, this probability will not change so long as we keep it as our choice. At the moment when only two doors remain, the other door will have probability $\frac{n-1}{n}$. That is the probability that we win with SLM.

We also know that there will never be a moment in the game when a door has a probability smaller than $\frac{1}{n}$. Thus, at the moment when only two doors remain it is impossible to produce a door with probability greater than $\frac{n-1}{n}$. This shows that SLM is optimal.

Very nice. A full, rigorous proof that SLM is, in fact, uniquely optimal can be found in [11]. You might also wonder what can be said about other strategies. For example, what if we are playing with fifty doors and we are absolutely determined to switch exactly seven times during the game? What is our best strategy? A consideration of such questions can be found in [8].

It would seem that a bit of clear-thinking can steer us through even the densest of Monty-inspired forests. Once our intuition has been tuned to what is important, it is not so difficult to ferret out the correct answer.

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