A fractal structure in cubic graphs

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Thanks to: Jerzy Filar, Vladimir Ejov, Jessica Nelson

A Numerical Experiment

A graph G has an associated adjacency matrix A, where

$$a_{ij} = \begin{cases} 1, & \text{if an edge joins vertices } i,\ j, \\ 0, & \text{otherwise.} \end{cases}$$

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Find the adjacency matrices of **all** cubic graphs with a given number of vertices. For each graph's matrix:

- Divide the matrix by 3, *stochastic matrix*
- Find its eigenvalues,
- Take their exponential, *otherwise mean zero*
- Find their mean and variance, *for statistical analysis*
- Plot a single dot of mean versus variance.

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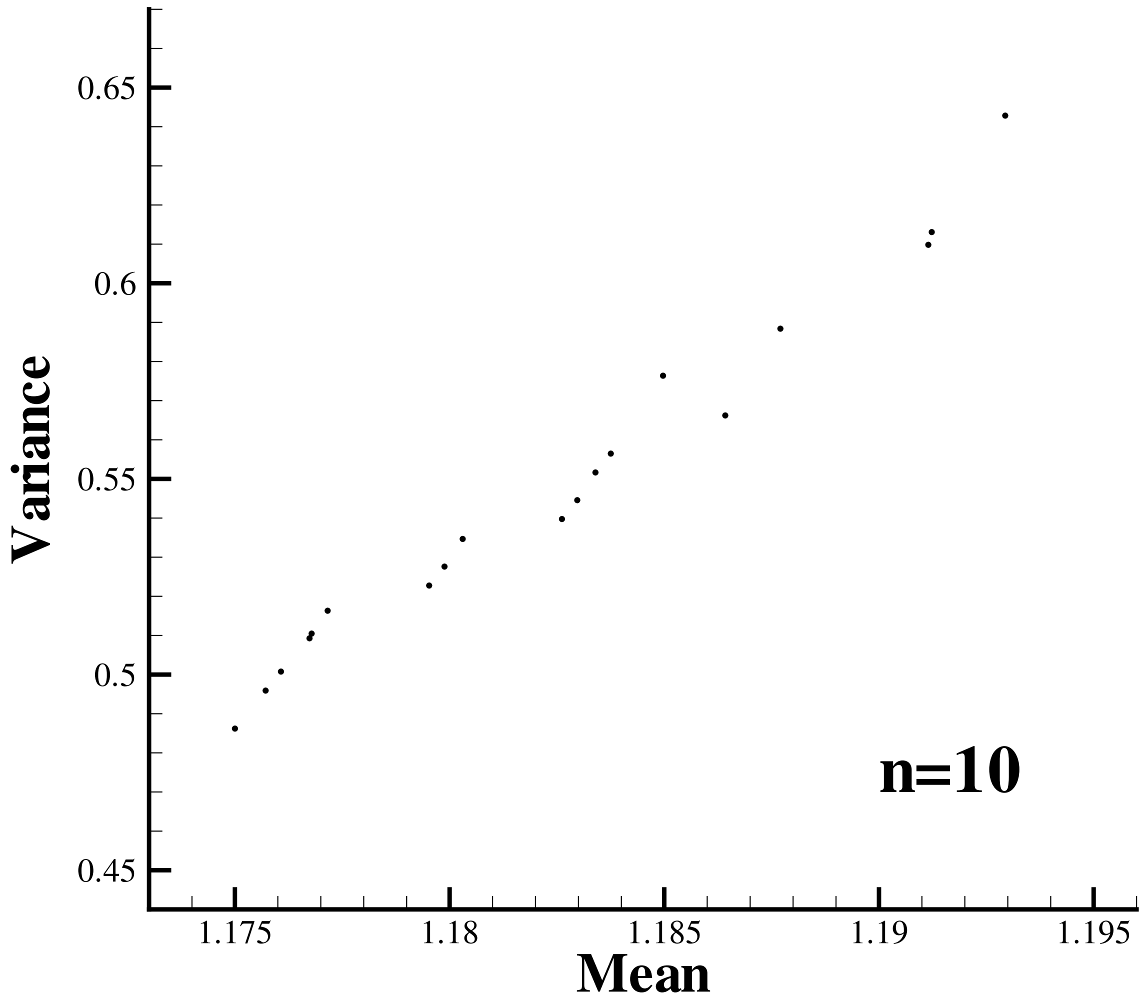
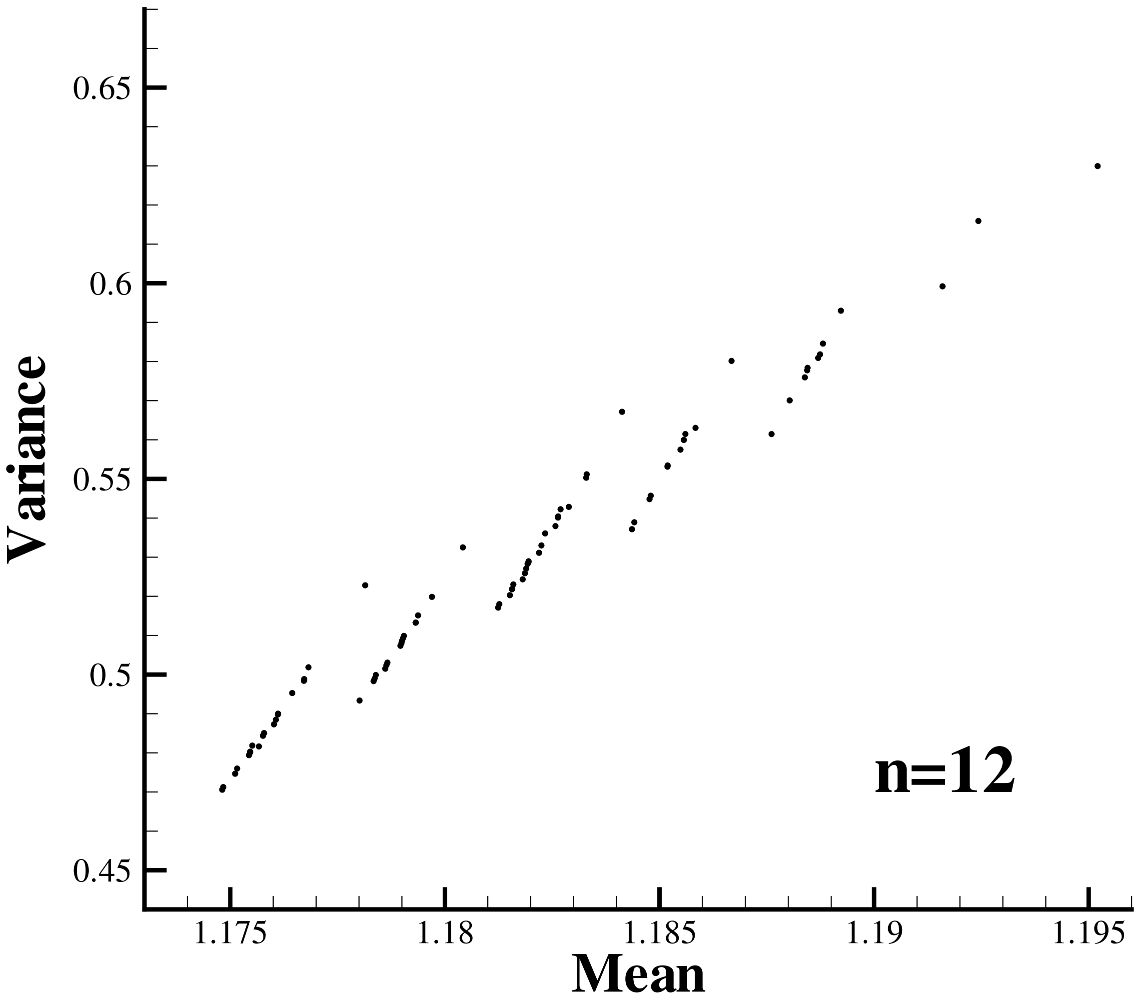
n	$\#G$
10	19
12	85
14	509
16	4060

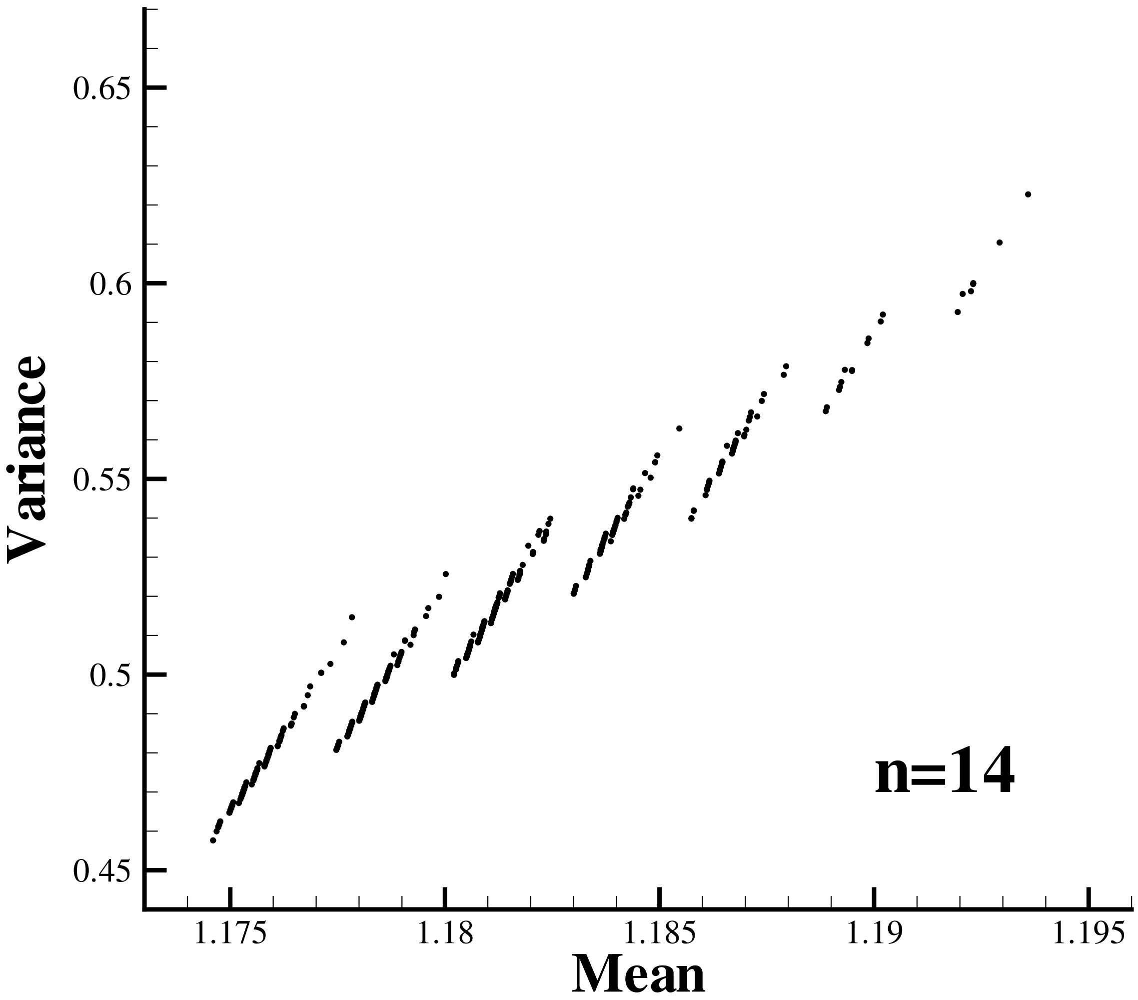
$n = 10$



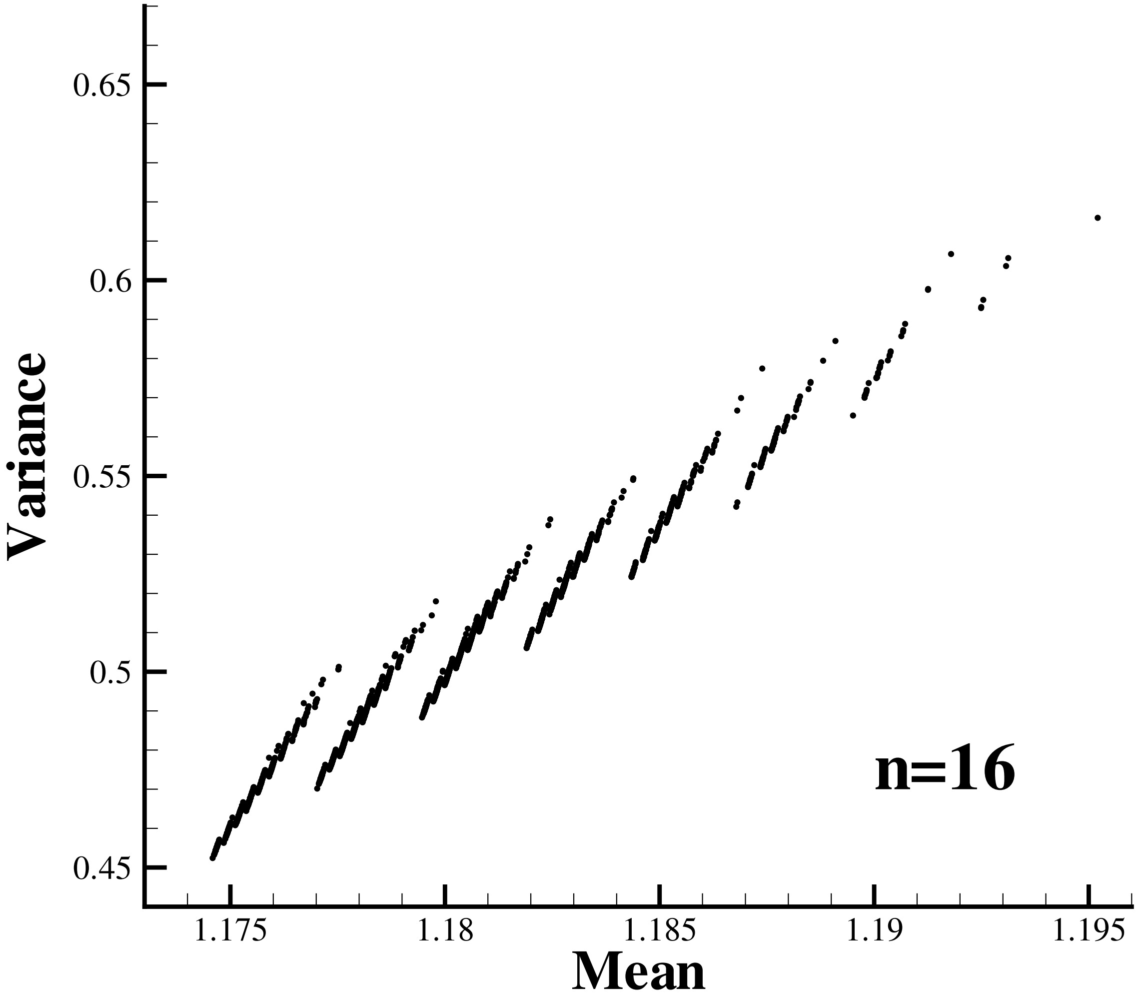
$n = 12$



$n = 14$



$n = 16$



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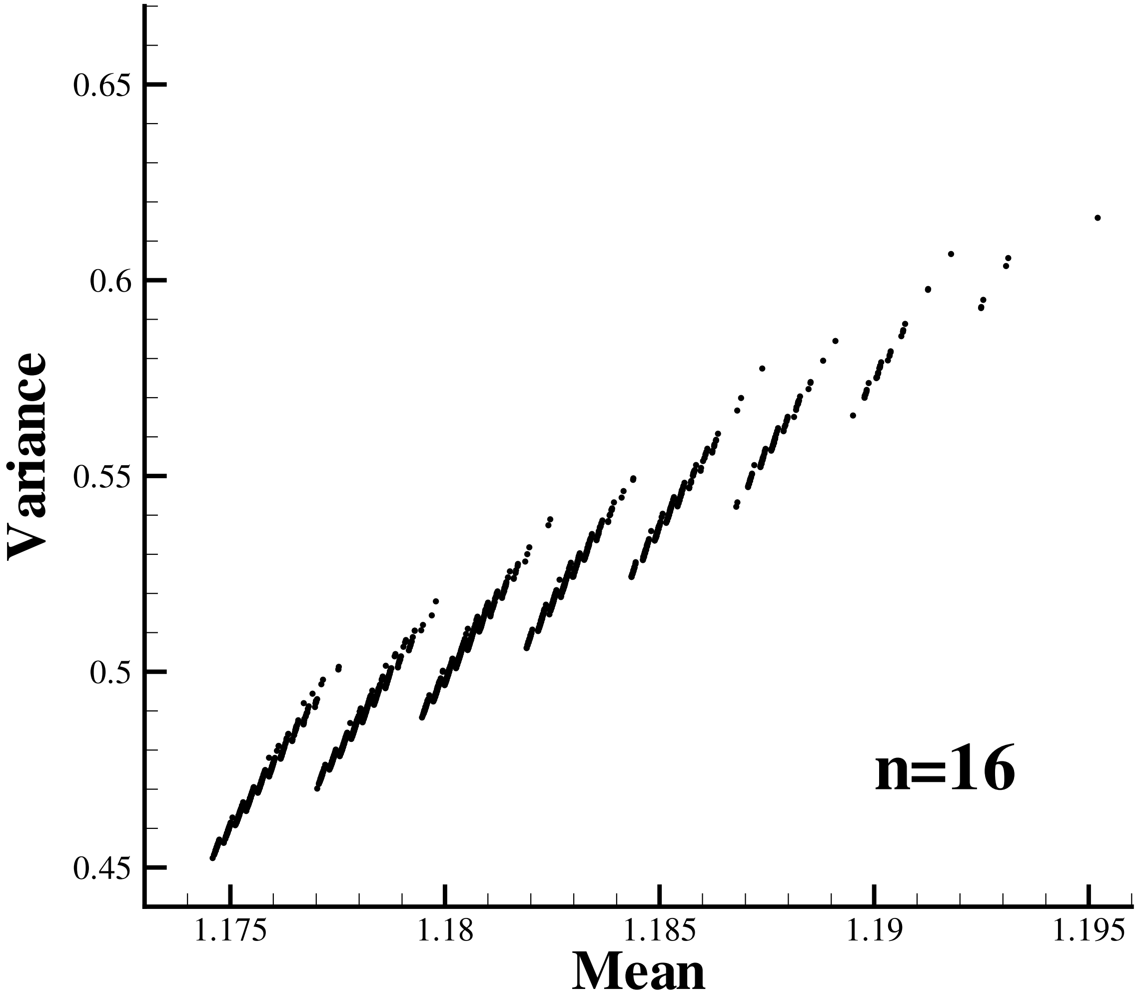

Data appears to be straight lines, with roughly the same slope and distance between them. Call them “filars” (threadlike).