MATH 235 (SPRING 2014) FINAL EXAM

MAY, 2014

Name: **Name:**
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Attempt all problems. Box your answers.

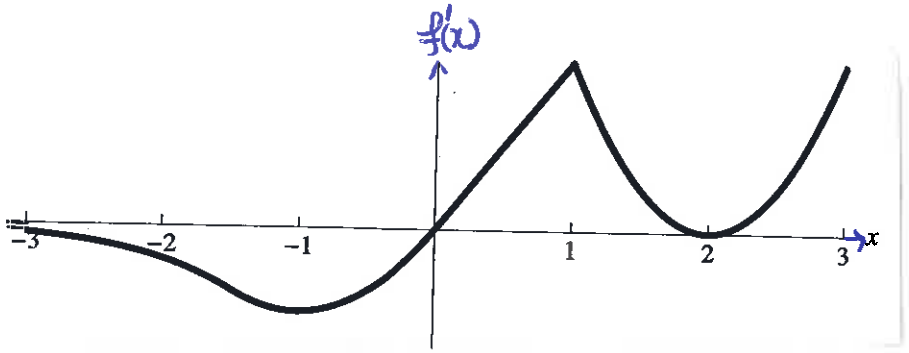
(1) A spherical melon has weight proportional to its volume ($w = kV$) as it grows. When the melon weighs 0.2 lbs, it has a volume of $36cm^3$, and its weight is increasing at a rate of 0.1 lbs per day. (The volume of a sphere is $V = \frac{4}{3}\pi r^3$).

2 (a) Find the rate of change of the volume of the melon when its weight is 0.2 lbs.

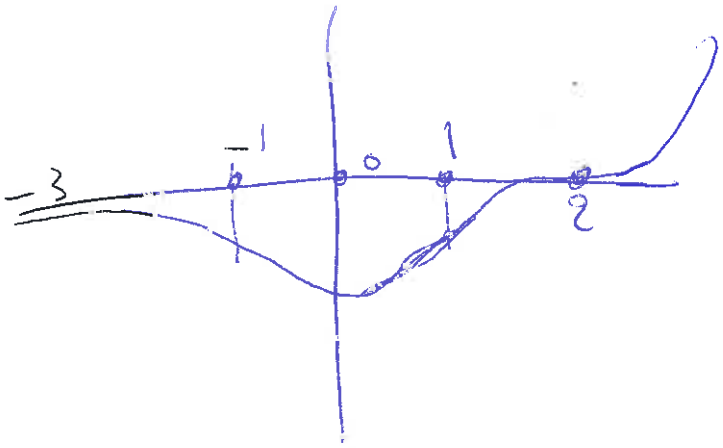
2 (b) Find the rate of change of the radius of the melon when its weight is 0.2 lbs.

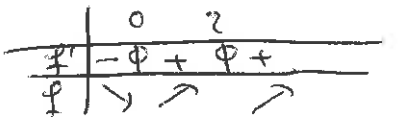
(c) Approximate the volume of the melon 36 hours after it weighs 0.2 lbs.

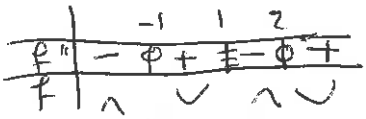
(2) Given below is a graph of $f'(x)$, the derivative of the function $f(x)$.



2 (a) Sketch a possible graph of $f(x)$.







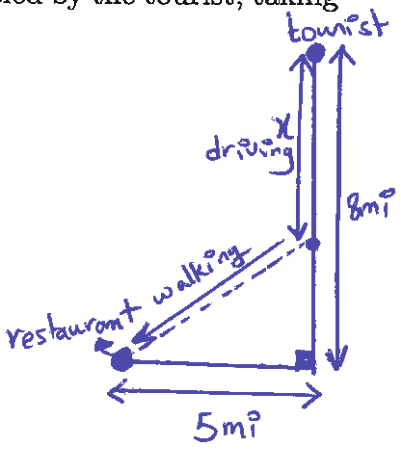
1 (b) List the x-coordinates of all inflection points of f. $x = -1, 1, 2$

1 (c) Give the x-coordinate of the global minimum of f on $[-3, 3]$. $x = 0$

1 (d) Give the x-coordinate of the global maximum of f on $[-3, 3]$. $x = 3$

(3) A tourist in Italy is driving his car south along a straight road towards a nearby restaurant. The restaurant is 8 miles south and 5 miles west of the car. The tourist plans to drive south for a while, park his car somewhere on the road, then walk the rest of the way to the restaurant (see the figure below). Assuming that the car speed is 4 times the tourist's walking speed, how far down the road should he park the car in order to get to the restaurant in the fastest way possible.

(Hint: Write down the function for the effective distance traveled by the tourist, taking his change of speed into account).



(4) A population of rodents is changing at a rate of $P'(t) = \frac{1}{(100-t)^2}$, where $P(t)$ is the number of rodents at time t, and t is measured in years.

(a) Find the net change in the population after 50 years.

(b) Find the net change in the population after T years.

(c) If the initial population is 20 rodents, use the previous part to write down $P(T)$ as a function of T.

(d) What happens to the population after a hundred years? (Hint: take $\lim_{T\to 100} P(T)$). Is this population realistic? Why?

(e) Accurately plot the graph of $P(T)$.

(5) The following function F represents the savings per year (in dollars) that Elina makes vs the cost x (in hundreds of dollars) of installing energy efficient equipment in her apartment.

$$F(x) = \begin{cases} 2e^{0.4x} & 0 \leq x \leq 5 \\ \dfrac{5x^2}{ax^b + c} & x > 5. \end{cases}$$

(a) Given that $F(x)$ is continuous, and that annual savings stabilize at 100 dollars as the equipment and installation costs increase, find a, b, and c.

3

(b) If Elina spent 700 dollars on installing energy efficient equipment, how long will it take for her savings to pay off the cost of installation?

2

(6) Find the following limits or integrals.

(a) $\lim_{x\to 1} \frac{\sin(\ln x)}{x-1}$.

1

(b) $\lim_{x\to\infty} x^{\frac{1}{x}}$.

2

(c) $\int e^x(1-e^{3x})dx$.

1

(d) $\int 3x^2 \sin(x^3-1)dx$.

1

(7) Cardiology: The following table gives the dye concentration (for a dye concentration cardiac-output determination) seconds after injection. The amount of dye injected in this patient was 5 mg. (Cardiac output is the volume of blood being pumped by the heart.)

seconds after injection (t)	2	4	6	8	10	12	14	16	18	20	22	24
Dye concentration in mg per liter (c(t))	0	0.6	1.4	2.7	3.7	4.1	3.8	2.9	1.7	1.0	0.5	0

(a) Plot an approximate graph of the dye concentration $c(t)$ against t. Why is the concentration zero at the beginning and zero at the end of the time interval?

1

(b) Use a left point Riemann sum to estimate the area under the dye concentration curve.

2

(c) Estimate the patient's cardiac output in liters per minute by dividing the amount of the initial injection by the area you just computed. (After you find the answer, perform a unit analysis to justify that this division indeed gives the units of the cardiac output: liters per minute.)

1

(d) An average resting cardiac output would be 5.6 liters/min for a human male, and 4.9 liters/min for a human female. Is this patient's cardiac output close to the average?

1