

MATH 235 (SPRING 2014) MIDTERM EXAM

WED MAR 5, 2014

Name:

Attempt all problems. Box your answers.

(1) Find the following limits:

(a) $\lim_{x \rightarrow -\infty} e^x \tan^{-1} x$

(b) $\lim_{x \rightarrow 0} \frac{\sin 2x}{4x}$

(c) $\lim_{x \rightarrow \infty} (\sqrt{x} - x)$

(2) Solve the inequality $|3x - 4| < \frac{1}{2}$.

- (3) (a) Prove that $\lim_{x \rightarrow 2^+} \frac{1}{x-2} = \infty$.
- (b) Plot the function $\frac{1}{x-2}$ and illustrate the above proof on your graph.

(4) Consider the function

$$f(x) = \begin{cases} x - 1 & \text{if } x \leq 1 \\ \ln(x - 1) & \text{if } x > 1 \end{cases}$$

(a) Find the interval on which f is continuous, and describe the type of any discontinuities it has.

(b) Plot the graph of f .

(c) Choose an interval where the intermediate value theorem for continuous functions applies, in order to deduce that f has a root on that interval. What is that root?

- (5) (a) Plot the graph of the rational function $g(x) = \frac{(x+1)(x-2)(x-3)}{(x-2)^2(x-3)}$. Clearly label on your graph any horizontal or vertical asymptotes, roots, and holes.

- (6) Find the domain of definition of the function $\frac{\sqrt{x-3}}{(x-5)^2}$.
- (7) Plot the graph of the function $\tan^{-1}(x) - \pi$. Specify the domain and the range that you chose for your function.