MATH 236 (FALL 2014) MIDTERM EXAM

THURS OCT 9, 2014

Name: **Name:**
Name: **Name:**

Attempt all problems. Box your answers.

(1) In one of Seinfeld's episodes, Kramer sues the coffee franchise for serving the coffee too hot. This was based on an actual lawsuit by an elderly woman who spilled a cup of hot McDonald's coffee on her lap. This problem investigates the temperature of hot coffee.

A cup of coffee at $90°C$ is put in a $20°C$ room. The coffee's temperature is changing at a rate of $r(t) = -7(0.9)^t \; °C/min$. Estimate the coffee's temperature after 10 minutes.

(2) The population of the world t years after 2010 is predicted to be $P(t) = 6.9e^{0.012t}$ billion.
   (a) What population is predicted in 2020?
   (b) What is the predicted *average* population between 2010 and 2020?

(3) Find the following integrals
   (a) $\int t \sin t dt$

(b) $\int \frac{y^2+2y+1}{(y^2+1)^2} dy$

(c) $\int_0^{\sqrt{3}/2} \frac{4x^2}{(1-x^2)^{3/2}} dx$

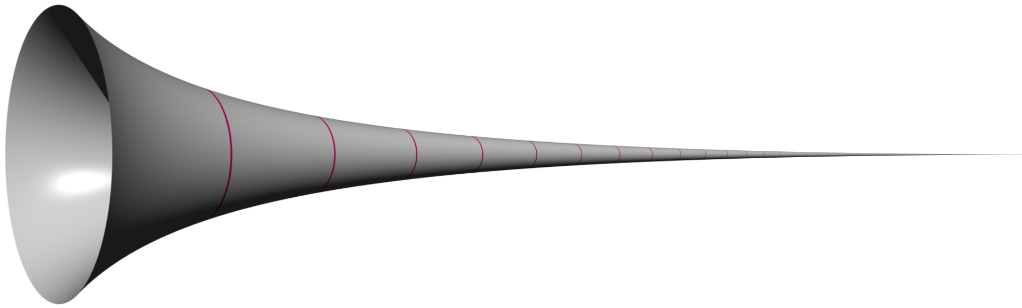

FIGURE 1. Gabriel's Horn

(4) Find the values of p for which the integral $\int_1^2 \frac{dx}{x(\ln x)^p}$ converges.

(5) (*Gabriel's Horn*) In the Abrahamic traditions, Archangel Gabriel blows an infinite horn (infinite for the divine) to announce judgement day.

Consider the solid obtained by revolving the area between the $x-axis$ and the function $f(x) = \frac{1}{x}$ on the interval $[1, \infty)$ about the $x-axis$. The surface of this infinite solid is called Gabriel's Horn, or The Infinite Paint Can (see Figure 1.)

(a) Find the volume of this infinite solid.

(b) Set up an integral that represents the surface area of this solid of revolution. (Do not compute the integral.)

(c) Use comparison between $\int_1^\infty \frac{1}{x}dx$ and your integral in part (b) to study the convergence or divergence of your integral.

(d) Use parts (a) and (c) to explain the following apparent paradox: "This is a can that does not hold enough paint to cover its own interior!".

(6) Find the hydrostatic force exerted on a damn in the shape of an isosceles triangle whose top is 200 ft wide and whose total height is 100 ft, given that the damn is holding back a body of water that reaches to the top of the damn.

(7) Write an integral that evaluates the volume of the solid obtained by revolving the region bounded by $f(x) = x^2 + 1$ and the $x - axis$ on the interval $[-1, 3]$ around the line $x = -1$. Do not evaluate the integral.

(8) Consider the function $f(x) = \sin x$.
   (a) Plot the graph of $f(x)$ on the interval $[0, \pi]$.
   (b) Discretize $[0, \pi]$ using $n = 3$ intervals, then approximate the arc length of the graph of $f(x)$ on $[0, \pi]$.
   (c) Write an integral that evaluates exactly the arc length of $f(x)$, then find the error in your approximation in part (b). (Evaluate the integral using a calculator or your smartphone.)