MATH 236 (FALL 2014) MIDTERM REVIEW SHEET

OCT 8, 2014

(1) **Fundamental Theorems of Calculus** (See Calc I review sheet) (one example in Quiz I).

(2) **Net Change Application of Fundamental Theorem of Calculus**
quantity at time $t =$ quantity at time $a + \int_a^t$ (rate of change of quantity)dt
which is equivalent to $f(t) = f(a) + \int_a^t f'(s)ds$ or $f(t) - f(a) = \int_a^t f'(s)ds$.
Example The population of Tokyo grew at a rate shown in the figure. Estimate the net change in Tokyo population between 1970 and 1990.

(3) **Integration Techniques** (substitution, by parts, partial fractions, trigonometric integrals, trigonometric substitutions, reappearance act).
Examples:
 (a) $\int \frac{8x^2+8x+2}{(4x^2+1)^2}dx$
 (b) $\int_0^1 \frac{dx}{(4-x^2)^{3/2}}$

(4) **Improper Integrals** (identifying the source of the improperness (singularity), set up limit representation, computation, convergence and divergence using comparisons to known integrals (such as $\int \frac{dx}{x^p}$) or the limit comparison test. *Examples:*
 (a) Find the values of p for which the following integrals converge $\int_2^\infty \frac{dx}{x(\ln x)^p}$; $\int_0^1 x^p \sin \frac{1}{x}dx$.
 (b) This example shows that $\int_{-\infty}^{\infty} f(x)dx$ is not necessarily equal to $\lim_{b\to\infty} \int_{-b}^{b} f(x)dx$. Show that $\int_0^\infty \frac{2xdx}{x^2+1}$ diverges and hence that $\int_{-\infty}^{\infty} \frac{2xdx}{x^2+1}$ also diverges, while, $\lim_{b\to\infty} \int_{-b}^{b} \frac{2xdx}{x^2+1} = 0$! Plot a graph of the function $\frac{2x}{x^2+1}$ and see whether you can have an intuition of what's going on (Hint: There are cancelations that are not allowed when you are considering infinite integrals (the rigorous answer involves absolute convergence)).

(5) **Applications of Integrals**
 (a) *Average of a function* Example: The population of McAllen, Texas, can be modeled by the function
$$P(t) = 570(1.037)^t,$$
where P is in thousands of people and t is in years since 2000. Use this function to predict the average population of McAllen between the years of 2020 and 2040.
 (b) *Area between two curves or signed area between the graph of a function and the $x-axis$*
Example: Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
 (c) *Volumes by Slices or Volumes by Shells* (mostly –but not restricted to (see the Giza Pyramid problem)– of surfaces of revolution around a horizontal axis or a vertical axes).
 (d) *Surface area of revolution*: $2\pi \int_a^b f(x)\sqrt{1+f'(x)^2}dx$ (derivation using surface area of frustum.)
 (e) *Arc length of $f(x)$*: $\int_a^b \sqrt{1+f'(x)^2}dx$, and estimating the arc length using discretization of the domain of $f(x)$. Going back and forth between discrete and the continuum using limits (derivation of the formula).
 (f) *Centroid of the region under the graph of $f(x)$* and the region between two graphs $f(x)$ and $g(x)$.
 (g) *Mass, Work, and Hydrostatic Force.*

(6) Later, the beautiful theory of **Differential Equations, and modeling** the world!