MATH 236 (FALL 2014) QUIZ I

THURS SEPT 24, 2014

Name: **Name:**
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Attempt all problems. Box your answers.

(1) Compute $\int (\ln x)^2 dx$ (Hint: Integration by parts twice). Find the exact value of the average of the function $f(x) = (\ln x)^2$ over the interval $1 \leq x \leq 4$.

(2) Find the following:

(a) $\int (\sin x \sqrt{\cos x})^3 dx$

(b) $\frac{d}{dx} \int_1^{\tan x} \frac{1}{t^2\sqrt{t^2+1}} dt.$

(c) $\int \frac{x^3+4x^2-21}{x^2+6x+10}dx.$

(d) $\int_1^3 \frac{1}{x^2(x+1)} dx$

(3) Frank is evaluating electric motors to drive automated mixing for some waste tanks that he must maintain. One pump is advertised to have a probability that follows the exponential distribution

$$f(t) = 0.31e^{-0.31t},$$

where the time $t > 0$ is measured in years. Frank knows that the expected time of failure for something following this distribution is

$$\int_0^\infty tf(t)dt.$$

How long can he expect one of these pumps to last?

(4) (a) Plot the graph of the function e^{-x^2}.
(b) Graphically, explain why $\int_{-1}^{1} e^{-x^2}dx > 1$.
(c) Give an estimate for $\int_{1}^{\infty} e^{-x^2}dx$ by comparison to the integral $\int_{1}^{\infty} xe^{-x^2}dx$.
(d) Use parts (b), (c) and your graph in part (a) to estimate $\int_{-\infty}^{\infty} e^{-x^2}dx$.
(e) Find the actual value $\int_{-\infty}^{\infty} e^{-x^2}dx$ using a calculator or your smart phone. (In Calc III, you will be able to evaluate this integral).

(5) For which values of p does the improper integral $\int_0^1 x^p \sin\frac{1}{x} dx$ converge? (Hint: It may be easier to use the substitution $u = \frac{1}{x}$, then $du = \ldots$. Then compare your new integral (with new boundaries) to an integral that you know. Make sure that your answer covers all possible values of p).