# MATH 236 (FALL 2014) QUIZ II ON CHAPTER 7

THURS NOV 6, 2014

**Name:** **Name:**
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**Attempt all problems. Box your answers.**

(1) Study the convergence of the following series. Specify the type of convergence for series with positive and negative terms (conditionally or absolutely convergent). Justify all your answers.

(a) $\sum_{n=3}^{\infty} \frac{\sqrt{n+2}-\sqrt{n-2}}{n}$

(b) $\sum_{n=1}^{\infty} \left(\sin \frac{1}{n}\right)^{1+\epsilon}$ where $0 < \epsilon < 1$.

(c) $\sum_{n=3}^{\infty} \frac{2}{n \sqrt[n]{n} \ln n}$

(d) $\sum_{n=1}^{\infty} \frac{\cos^2(n)}{n^{3/2}}$

(e) $1 - \frac{1}{2} - \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} - \frac{1}{7} - \frac{1}{8} - \frac{1}{9} - \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \frac{1}{13} + \frac{1}{14} + \frac{1}{15} - \dots$

(2) **Fractal: Koch snowflake** In Figure 1, the area inside the Koch snowflake can be described as the union of infinitely many equilateral triangles. Each side of a smaller triangle is exactly one third the size of a side of the large one. If the area if the inner most triangle is 1 unit of area, find the total area of the Koch snowflake. (The area of an equilateral triangle is $\frac{\sqrt{3}}{4}(side)^2$. Note that the area of each triangle in the new layer is $\frac{1}{9}$th the area of the one in the previous layer. You also need to count how many triangles are added while constructing each new layer.)

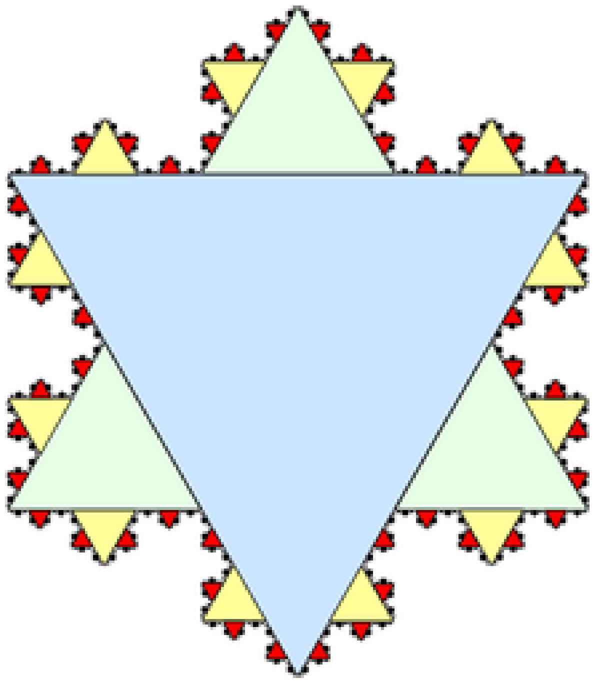

FIGURE 1. Koch snowflake (source: Wikipidea)

(3) Prove that the series $\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^2}$ is convergent, and estimate the answer up to 6 accurate digits (you want to use enough terms in the finite sum so that the error due to truncation is less than $10^{-6}$). *Hint:* Use the error bound from the integral test.

(4) **Fixed point iteration:** We will find a root of the function $f(x) = x^4 - x - 10$ using a fixed point iteration instead of Newton's method (saves us the trouble of differentiating $f(x)$).

We want to solve $f(x) = 0$ which we can write as $x = (x + 10)^{\frac{1}{4}}$ or $x = g(x)$ where $g(x) = (x+10)^{\frac{1}{4}}$. Hence, a root of $f(x)$ is a *fixed point* of $g(x)$. Just like we did in Newton's method, we will construct a sequence of numbers that hopefully will converge to a fixed point of $g(x)$, which will be a root of $f(x)$.

(a) Start with initial guess $x_0 = 1$.
(b) Construct a sequence using the recurrence relation $x_{k+1} = g(x_k)$, for $k = 0, 1, 2, \ldots$.
(c) Stop when $|x_{k+1} - x_k| < tolerance$, where tolerance$= 10^{-5}$.
(d) Check that your last iterate $x_m$ is indeed a fixed point of $g(x)$ (so $x_m \simeq g(x_m)$) and hence a root of $f(x)$ (so $f(x_m) \simeq 0$).

(5) (a) Consider the sequence $\{\sin \frac{n\pi}{6}\}$.

(i) Is it bounded? What are the least upper bound and the greatest lower bound?

(ii) Is it convergent? Does it have any convergent subsequences? How many subsequential limits can you identify? What are they?

(iii) Is it monotonic? Why or why not?

(iv) Does it have a divergent subsequence?

(b) Is the corresponding series $\sum_{n=1}^{\infty} \sin \frac{n\pi}{6}$ convergent? Why or why not?