MATH 236 (FALL 2014) QUIZ III ON CHAPTER 8

THURS DEC 4, 2014

Name: **Name:**
Name: **Name:**

Attempt all problems. Box your answers.

(1) Consider the series

$$1 - x + x^2 - x^3 + \ldots = \sum_{n=0}^{\infty} (-1)^n x^n.$$

(a) Find the radius of convergence, interval of convergence, and the type of convergence on that interval.

(b) The above series is also a geometric series. What is the common ratio r? What is the exact sum of the above series? (Hint: Use the formula $\frac{r^{\text{first index}}}{1-r}$).

(c) Deduce from part (b) the identity

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$$

and find the interval on which it is valid (check the endpoints individually).

(d) How many terms from the above series do you need to use to be able estimate $\ln(2)$ to within 0.001 of its value? What is that value?

(e) Plot a (very neat) graph of $\ln(1+x)$, its Taylor polynomial of order 5, and of its series expansion (all on the same graph). Specify clearly the interval on which the series is well-defined.

(2) Write down the definition of:

(a) (i) f is smooth on the interval (a, b).

(ii) f is analytic on the interval (a, b).

(b) Is every real valued smooth function analytic?

(c) Show that the function e^{-1/x^2} and its Taylor series expansion around $x_0 = 0$ disagree everywhere except at $x = 0$. Plot a figure to illustrate. Is this function analytic anywhere?

(3) (a) Prove that the function $\frac{\sin x}{x}$ is analytic on $\mathbb{R}$ and find its Taylor expansion around $x_0 = 0$.

(b) Plot $\frac{\sin x}{x}$ and its Taylor polynomial approximations of order 6 and 8, on the interval $(-10, 10)$.

(c) What must you change if we were to consider the function $\frac{\cos x}{x}$ instead?

(4) Find all values of x for which the following series converges

$$\sum_{n=0}^{\infty} \frac{x^n}{1+n2^n}.$$

When is the convergence absolute and when is it conditional?

Review for final, do not solve Using the Lagrange form of the Taylor remainder $R^n(x) = \frac{f^{(n+1)}(c)(x-a)^{n+1}}{(n+1)!}$, prove that

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots.$$