Math 236 Calculus II (Fall 2014)

Calculus I (Math 235) Basics Review Sheet

Many topics were covered in Calculus I. The following is a review for only the *basic tools and ideas in CalcI* and you should master them *before* taking Calculus II.

H.w. Assignment 1: Solve all the numbered problems, write them very neatly, and submit them in class on Thursday August 28 2014.

1 Functions of One Variable

In Calculus I, we encountered the following **types of functions**:

- polynomial functions (e.g. $x^3 - \frac{1}{3}x + 5$, $2x - 1$),
- rational functions (e.g. $\frac{2x^3-0.5}{x+4}$),
- algebraic functions (e.g $\sqrt[3]{x^2-1}$),
- trigonometric functions (e.g. $\sin x$, $\cos x$, $\tan x$, $\cot x$, $\sec x$, $\csc x$),
- inverse trigonometric functions ($\arcsin x$, $\arccos x$, $\arctan x$, $\sec^{-1} x$) (restrict to an interval where the original trigonometric function is invertible i.e. one-to-one, or passes the horizontal line test, then define the inverse trigonometric function),
- exponential functions (e.g. e^x, 2^x, $\left(\frac{1}{2}\right)^x$),
- logarithmic functions (e.g. $\ln x$, $\log_b x$).

(**Review the domains of definitions** (denominator $\neq 0$, under the square root ≥ 0, inside the logarithm > 0, domains and ranges of inverse trigonometric functions, etc.), and **careful plotting** of these functions.)

1. Find the domains of definition of the following functions:

   (a) $f(x) = \ln\left(\sqrt{\frac{x-1}{x^2-2x-3}}\right)$

   (b) $g(x) = (0.5)^{x^3-1}$

   (c) $h(x) = \sqrt{\sec x}$

   (d) $m(x) = 3\arcsin(x-4)$

2. Match the functions 2^x, x^3, $\frac{\ln x}{\ln 8}$, $x^{1/3}$ (which is the same as $\sqrt[3]{x}$) to their graphs in Figure 1. Evaluate each of the four functions at $x = 13$ and explain the order that you see in your answer. Is this order reflected in the given graph? Explain.

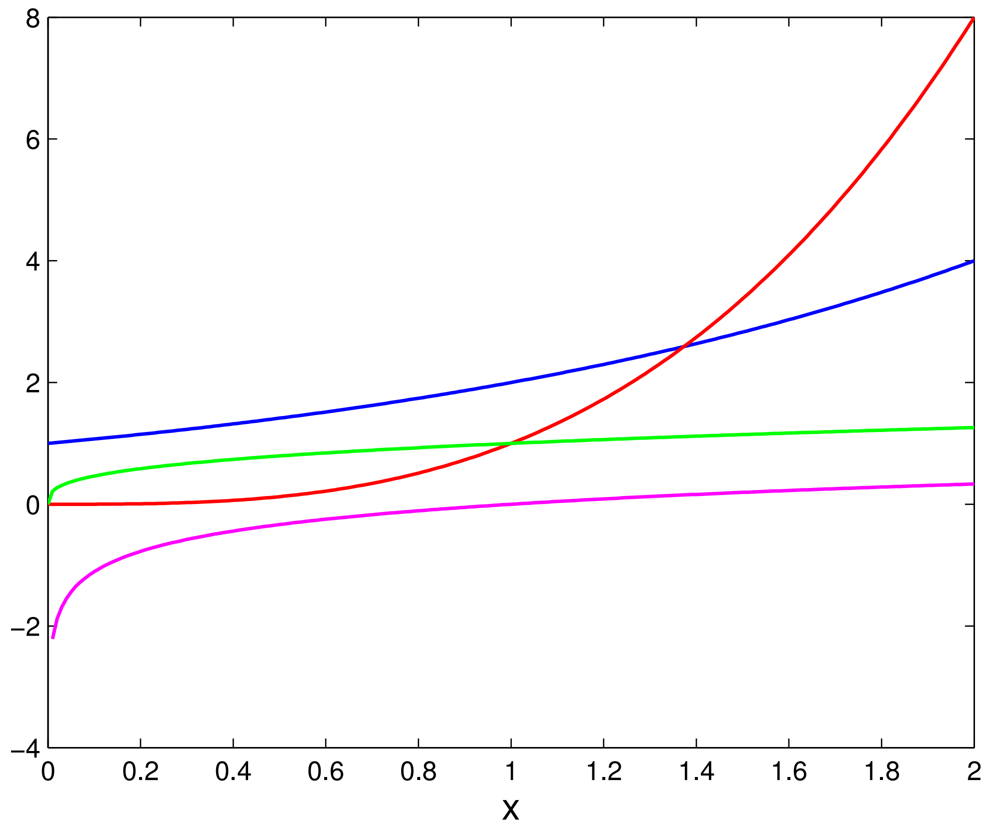

Figure 1: Match the functions 2^x, x^3, $\frac{\ln x}{\ln 8}$, $x^{1/3}$ to their graphs.

2 Limits

Find the following limits (you may need **L'Hospital** rule to compute some of them). Recall the log**, polynomial, and exponential power struggle**.

1. $\lim_{x\to-\infty} 2x^3 + x^2 - 7$
2. $\lim_{x\to-2} \frac{4+2x}{x^2+2x}$
3. $\lim_{x\to0} \frac{3\sin x+x}{x}$
4. $\lim_{x\to-\infty} e^x \tan^{-1} x$
5. $\lim_{x\to0} \frac{\sin 2x}{4x}$
6. $\lim_{x\to\infty}(\sqrt{x} - x)$
7. $\lim_{x\to1} \frac{\sin(\ln x)}{x-1}$.
8. $\lim_{x\to\infty} x^{\frac{1}{x}}$.

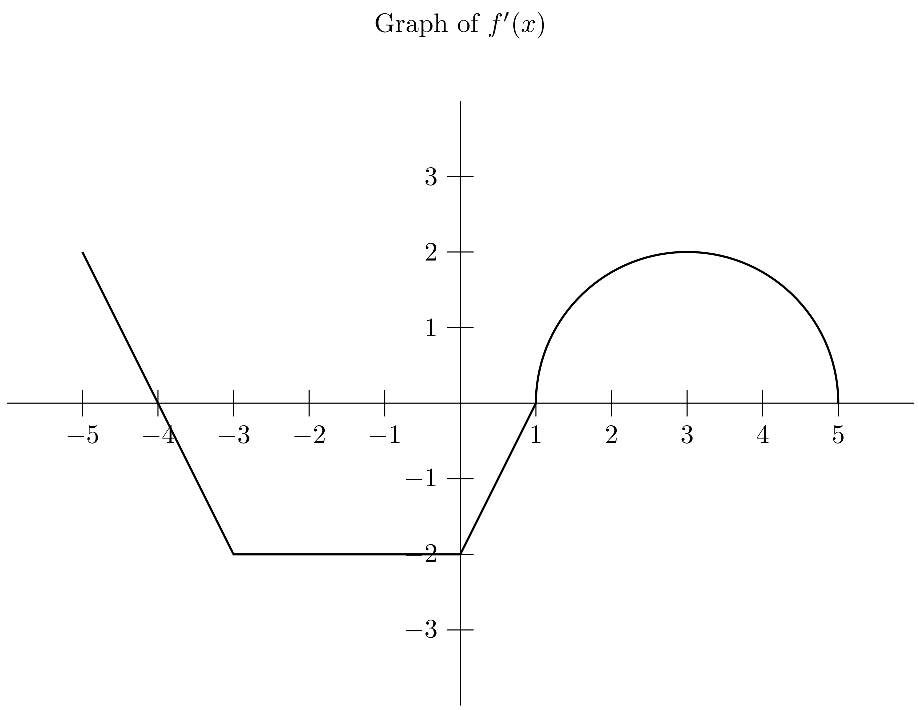

Figure 2: Sketch a graph of $f(x)$.

3 Derivatives

Review chain rule, implicit differentiation, logarithmic differentiation

1. Compute the following derivatives
   (a) $f(x) = \sqrt{x \ln(2^x + 1)}$
   (b) $g(x) = \frac{1}{x} \sin^3(2x)$
   (c) $m(x) = (x^2 - 3)^x$ (Hint: logarithmic differentiation: set $y = (x^2 - 3)^x$ then compute the derivative of $\ln y$ implicitly. Deduce y'.)

   Critical points of a function are when $f'(x) = 0$ or when $f'(x)$ does not exist. **Review first and second derivative tests to study the nature of critical points and locate any local extrema (maxima or minima) and inflection points of a function.**

2. Given in Figure 2 a graph of $f'(x)$, the derivative of a function $f(x)$. The graph is made up of lines and a semicircle.
   (a) Sketch a graph of $f(x)$ on the interval $[-5, 5]$.
   (b) List the x-coordinates of all inflection points of f.
   (c) Give the (x, y)-coordinates of the global minimum of f on $[-5, 5]$.
   (d) Give the (x, y)-coordinates of the global maximum of f on $[-5, 5]$.

Review mean value theorem, optimization problems and related rates problems.

4 Integrals

The fundamental theorem of calculus (Under the right conditions, derivatives and integrals 'undo' each other.)

(I) If f is a continuous function on $[a,b]$ and F is any antiderivative of f (so $F'(x) = f(x)$), then

$$\int_a^b f(x)dx = F(x)|_a^b = F(b) - F(a)$$

(II) (*Net Change*) If g is differentiable on $[a,b]$ then

$$\int_a^b g'(x)dx = g(x)|_a^b = g(b) - g(a)$$

(III) If f is continuous on $[a,b]$ and we define $F(x) = \int_a^x f(t)dt$ for all $x \in [a,b]$, then our new function $F(x)$ is continuous on $[a,b]$ and differentiable on (a,b), and $F'(x) = f(x)$ (so

$$\frac{d}{dx}\int_a^x f(t)dt = f(x)$$

for all $x \in [a,b]$).

Note: It follows by the chain rule that if $u(x)$ is differentiable on $[a,b]$ then

$$\frac{d}{dx}\int_a^{u(x)} f(t)dt = f(u(x))u'(x).$$

1. Find the following integrals
   (a) $\int e^x(1 - e^{3x})dx$.
   (b) $\int 3x^2 \sin(x^3 - 1)dx$.
   (c) $\int \frac{2-3xe^x}{x}dx$.
   (d) $\int_2^4 \frac{x^4+2}{x}dx$

2. Let $h(x) = \int_0^{2x} \frac{t^2-25}{1+\sin^2 t}dt$. Find all x at which $h(x)$ has a local maximum. (Hint: Use the fundamental theorem of calculus (III) above, the note that follows it, and the first or second derivative tests to locate local extrema (maxima or minima)).

3. A population of rodents is changing at a rate of $P'(t) = \frac{1}{(100-t)^2}$, where $P(t)$ is the number of rodents at time t, and t is measured in years.
   (a) Find the net change in the population after 50 years.
   (b) Find the net change in the population after T years.
   (c) What happens to the population after hundred years? Is this model realistic?