Math 336 Fall 2015

1^{st} order diff. eqns (linear, nonlinear) 2^{nd} order diff. eqns (linear, nonlinear) (homogeneous, nonhomogeneous) systems of diff. eqns (linear) systems of diff. eqns (nonlinear)	Analytical solns when possible Graphical representation Numerical methods & solns qualitative analysis: soln properties critical pts & stability analysis Applications

Week 1-2 Highlights

First Order Differential Equations

Types we encounter

1) $y' = f(t,y)$ (could be linear or nonlinear)

2) $y' = f(t)g(y)$ (separable) (could be linear or nonlinear)

3) $y' + p(t)y = q(t)$

or $a(t)y' + b(t)y = c(t)$, $a(t) \neq 0$

analytical soln: use Integrating factor. $\text{I.F.} = e^{\int p(t)dt}$

then $y(t) = \frac{1}{\text{I.F.}}\left[\int q(t)(\text{I.F.})dt + C\right]$

4) $M(x,y)dx + N(x,y)dy = 0$

- This is exact when $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

analytical soln: Find $f(x,y)$ such that $\frac{\partial f}{\partial x} = M$ and $\frac{\partial f}{\partial y} = N$

Then the soln is given by $f(x,y) = C$

- We can sometimes make a nonexact ODE exact by multiplying by an I.F. (which is not easy to guess usually).

Applications

1) Skydiving: without parachute, with parachute
external landing: without air resistance, with air resistance

2) Population prbs: Natural model, promiscuous model
The logistic eqn
doomsday vs extinction

Particular Solns and General solns

Due to integration constants [redacted] ODEs have infinitely many solns when they exist, but we can write the general form of the soln that includes these arbitrary constants.
The general soln of a 1^{st} order ODE has ONE arbitrary constant.

A particular soln is the soln that satisfies a certain condition (initial cond. at $t=0$, boundary conditions etc...)

Transforming a higher order ODE to a "system of 1^{st} order ODEs"

Trick: give new names to higher order derivatives.

Existence & Uniqueness of solns of $y' = f(t,y)$

(in Chapter 3) (solns of nonlinear ODEs may cease to exist in finite time)

Numerical solns

(in Chapter 8)

Matlab fn to solve $\begin{cases} y' = f(t,y) \text{ on } [t_0, t_f] \\ y(t_0) = y_0 \end{cases}$

Ode45

Graphical representation for $y' = f(t,y)$

- slope field: at each point in the t-y plane, draw a little segment with slope $f(t^*, y^*)$

To find a particular solns follow the path "suggested" by the slope field.

ODE soln
- analytical
- numerical
- graphical (approximate soln)

From Physics: Newton's second law of motion

$\underline{F} = m\underline{a}$

1