

- 1st order diff. eqns (linear, nonlinear)
- 2nd order diff. eqns (linear, nonlinear) (homogeneous, nonhomogeneous)
- systems of diff. eqns (linear)
- systems of diff. eqns (nonlinear)

Analytical solns when possible
 Graphical representation
 Numerical methods & solns
 qualitative analysis: soln properties
 critical pts & stability analysis
 Applications

Week 1-2 Highlights

First Order Differential Equations

Types we encounter

- $y' = f(t, y)$ (could be linear or nonlinear)
- $y' = f(t)g(y)$ (separable) (could be linear or nonlinear)
- $y' + p(t)y = q(t)$

or $a(t)y' + b(t)y = c(t), a(t) \neq 0$

analytical soln: use Integrating factor. I.F. = $e^{\int p(t)dt}$
 then $y(t) = \frac{1}{\text{I.F.}} \left[\int q(t)(\text{I.F.})dt + C \right]$

4) $M(x, y)dx + N(x, y)dy = 0$

• This is exact when $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

analytical soln: find $f(x, y)$ such that $\frac{\partial f}{\partial x} = M$ and $\frac{\partial f}{\partial y} = N$

Then the soln is given by $f(x, y) = C$

• We can sometimes make a nonexact ODE exact by multiplying by an I.F. (which is not easy to guess usually).

Applications

- Skydiving: without parachute, with parachute
or linear landing without air resistance, with air resistance
- Population probs: Natural model, promiscuous model
 The logistic eqn
 doomsday vs extinction

Particular Solns and General solns

Due to integration constants [redacted] ODEs have infinitely many solns when they exist, but we can write the general form of the soln that includes these arbitrary constants.
 The general soln of a 1st order ODE has ONE arbitrary constant.

A particular soln is the soln that satisfies a certain condition (initial cond. at $t=0$, boundary conditions etc...)

Transforming a higher order ODE to a "system of 1st order ODEs"

Trick: give new names to higher order derivatives.

Existence & Uniqueness of solns of $y' = f(t, y)$

(in Chapter 3) (solns of nonlinear ODEs may cease to exist in finite time)

Numerical solns

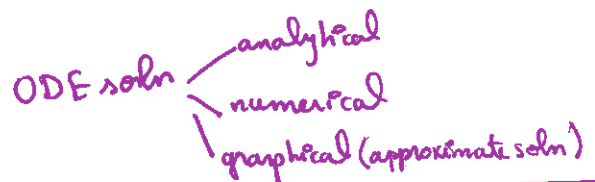
(in Chapters 8)

Matlab fn to solve $\begin{cases} y' = f(t, y) \\ y(t_0) = y_0 \end{cases}$ on $[t_0, t_f]$

Ode45

Graphical representation for $y' = f(t, y)$

• slope field: at each point in the $t-y$ plane, draw a little segment with slope $f(t, y)$
 To find a particular soln follow the path "suggested" by the slope field.



From Physics: Newton's second law of motion

$$\underline{F} = m\underline{a}$$