Math 336 Ordinary Differential Equations Midterm Exam

Fall 2015

Box your answers.

1 Solve the following ODEs or IVPs

If the ODEs admit complex solutions, write BOTH the complex solutions and the real solutions.

1. $y' = y^2 t^2$, $y(0) = 1$. Plot the solution.

2. $y' + y - 1 = 0$, $y(0) = 0$. Plot the solution.

3. $y' = \frac{-4x^3-4y}{4x-y^4}$. (Hint: Is this ODE exact?). Plot two distinct solution curves.

4. $y'' - 12y' + 36y = 0$, $y(0) = 4$ and $y'(0) = 0$. Plot the solution.

5. $y'' + 8y + 25y = 0$.

6. $y'' + 4y' + 5y = 5x + 5e^{-x}$.

7. $y'' - 6y' + 9y = e^{3x}$.

2 Solve the following to the best of your knowledge

1. **(Skydiver velocity model)** A skydiver jumps off an airplane and is in vertical downward motion. He deploys his parachute 15 seconds into his fall. Assuming that his initial velocity $v(0) = 0$, and that air resistance F_R is proportional to v^2 before parachute deployment, and to v after parachute deployment. Let $g = 32.2 ft/sec^2$, $k_1/m = 0.00104$ and $k_2/m = 2.01$, where m is the body mass of the skydiver, and k_1 and k_2 are the proportionality constants between F_R and v^2 and v respectively.

   (a) Write a first order ODE modeling the velocity of the skydiver. Do not solve the ODE.

   (b) What is the limiting velocity of the skydiver before and after deployment?

2. **Convert the following higher order ODE into a system of first order ODEs**

$$y'' - 3y' + 2y = 3e^x - 10\cos(3x).$$

Find the form of the particular solution of the above ODE (don't forget to check for duplication).

3 Numerical

Consider the IVP $y' = \frac{-2xy}{1+x^2}$, $y(0) = 1$.

1. Find the analytical solution and plot it on the interval $[0, 5]$.
2. Solve numerically using Euler's method with step size $h = 0.5$ on the interval $[0, 5]$. Plot the numerical solution on the same graph above.
3. Estimate the error using the second derivative of the exact solution.