Math 336 Ordinary Differential Equations

Fall 2015 Semester Summary

Types of ODEs that we now know how to deal with

1. **First Order ODEs in 1D**
   - **Existence and uniqueness of solutions**: Picard's theorem.
   - We have **analytical solutions** for:
     (a) Separable: Linear, nonlinear.
     (b) Exact: Linear, nonlinear. (Solution is represented by the level sets $f(x, y) = C$).
     (c) Linear, non-constant coefficients, non-homogeneous: Solve using integrating factor.
   - **Numerical solutions**: Euler, Improved Euler, and Runge-Kutta Methods. Programming, graphing, and error estimation.
   - **Visualizing**: Graphing analytical or numerical solutions. Direction Fields.
   - **Applications**: Falling bodies velocity models, population models (logistic equation, etc...).

2. **First Order Systems of ODEs**
   (a) **Existence and uniqueness of solutions**: Picard's theorem.
   (b) **Linear systems** We have analytical solutions for *linear* systems of ODEs with constant coefficients, homogeneous and non-homogeneous (eigenvalue-eigenvector method). Visualizing solutions: Graphing each coordinate as a function of t; phase portraits (depending on the eigenvalues and eigenvectors).
   (c) **Nonlinear systems** We never attempted finding analytical solutions, but we learned to: Find critical points and linearize near them, if the critical point is simple then the (qualitative) behavior of the nonlinear system can be predicted from the linearized systems near its critical points. Visualizing solutions: Graphing each coordinate as a function of t; phase portraits (including all critical points).
   (d) **Numerical solutions**: Same methods as ones we used for first order ODEs in 1D, careful that adding dimensions becomes computationally very expensive!
   (e) **Applications**: Ecological systems. Nonlinear pendulum (damped and undamped).

3. **Higher order linear ODEs in 1D**

   We only worked with LINEAR higher order ODEs.

   (a) **Existence and uniqueness of solutions**: Follows from Picard's theorem when all the coefficients involved are continuous.
   (b) **Representation of solutions**: Linear independence, Wronskian, general solution.
   (c) **Analytical solutions for**
   - Higher order constant coefficient linear ODEs:
     i. Homogeneous: Using characteristic equation (ansatz e^{rt}).
     ii. Non-homogeneous: complementary solution. Particular solutions: methods of undetermined coefficients and variation of parameters.
   - Second order non-constant coefficient linear ODEs:

i. Power series method- Taylor expansion when all coefficients are analytic near a point.

ii. Power series method-Frobenius series if the coefficients have singularities but these are regular singular points.

(d) **Visualizing** Graphing solutions as a function of t. Interpreting solutions for applications problems.

(e) **Applications** Mechanical vibrations: damped and undamped systems, transient solutions, steady state periodic solutions, resonance and practical resonance, *etc.*.

(f) Transforming into a system of first order ODEs (this can be done for both linear and nonlinear ODEs, as long as the ODE allows us to solve for the highest order derivative $y^{(n)}(t)$ in terms of the others).