# Math 336 Ordinary Differential Equations Written Assignment 1

First Order Differential Equations

## 1 Reading assignment

Read chapter 1 from the book.

## 2 Problem set (due Thursday September 17 2015)

1. **(Skydiver velocity model)** A skydiver jumps off an airplane and is in vertical downward motion. He deploys his parachute 15 seconds into his fall. Assuming that his initial velocity $v(0) = 0$, and that air resistance $F_R$ is proportional to $v^2$ before parachute deployment, and to $v$ after parachute deployment. Let $g = 32.2 ft/sec^2$, $k_1/m = 0.00104$ and $k_2/m = 2.01$, where $m$ is the body mass of the skydiver, and $k_1$ and $k_2$ are the proportionality constants between $F_R$ and $v^2$ and $v$ respectively.

   (a) Write a first order ODE modeling the velocity of the skydiver.

   (b) Solve your ODE. (Keep in mind the continuity condition at $t = 15 sec.$)

   (c) What is the limiting velocity of the skydiver before and after deployment?

2. **(Population model: doomsday vs extinction)** Consider a population whose birth rate increases at a rate proportional to itself, and whose death rate is constant.

   (a) Show that this population change can be modeled by the first order ODE $\frac{dP}{dt} = kP(P - M)$.

   (b) What are the critical points of this ODE? Are they stable? Unstable?

   (c) Make a $t$-$P$ plot illustrating the long term behavior of the population depending on different initial values $P_0$.

   (d) Why is this model called *doomsday vs extinction*?

   (e) Write down the analytical solution of the ODE with initial condition $P(0) = P_0$ and comment on $\lim_{t\to\infty} P(t)$. Contrast your answer with part (c) above.

## 3 Matlab assignment

1. **(Graphical solution- direction field)** Use DFIELD8 command in Matlab to plot the direction field of the ODE $y' = y^2 - 3t$. Plot the particular solution with initial condition $y(2) = 1$. Note that the trajectories do not cross (food for thought: why?). Using the methods for first order ODEs that we know so far, can you analytically solve the above ODE?

2. **(Numerical solution)** Use ODE45 command in Matlab to solve the above initial value problem ($y' = y^2 - 3t$ with $y(2) = 1$). Then use PLOT to plot your solution. Compare with the approximate solution you obtained using DFIELD8.