

**1. (1 pt) Library/274/Lin1stord/prob7.pg**

Solve the following initial value problem:

$$\frac{dy}{dt} + 0.5ty = 5t$$

with  $y(0) = 6$ .

$y =$  \_\_\_\_\_.

**2. (1 pt) Library/274/Lin1stord/prob2.pg**

Find the particular solution of the differential equation

$$\frac{dy}{dx} + y \cos(x) = 4 \cos(x)$$

satisfying the initial condition  $y(0) = 6$ .

Answer:  $y(x) =$  \_\_\_\_\_.

**3. (1 pt) Library/274/Exact/prob5.pg**

The differential equation

$$y + 2y^5 = (y^4 + 4x)y'$$

can be written in differential form:

$$M(x, y) dx + N(x, y) dy = 0$$

where

$M(x, y) =$  \_\_\_\_\_, and  $N(x, y) =$  \_\_\_\_\_.

The term  $M(x, y) dx + N(x, y) dy$  becomes an exact differential if the left hand side above is divided by  $y^5$ . Integrating that new equation, the solution of the differential equation is \_\_\_\_\_ =  $C$ .

**4. (1 pt) Library/274/Exact/prob3.pg**

Use the "mixed partials" check to see if the following differential equation is exact.

If it is exact find a function  $F(x, y)$  whose differential,  $dF(x, y)$  gives the differential equation. That is, level curves  $F(x, y) = C$  are solutions to the differential equation:

$$\frac{dy}{dx} = \frac{-4x^3 - 4y}{4x - y^4}$$

First rewrite as

$$M(x, y) dx + N(x, y) dy = 0$$

where  $M(x, y) =$  \_\_\_\_\_,

and  $N(x, y) =$  \_\_\_\_\_.

If the equation is not exact, enter *not exact*, otherwise enter in  $F(x, y)$  as the solution of the differential equation here \_\_\_\_\_ =  $C$ .

**5. (1 pt) Library/274/Exact/prob1.pg**

Use the "mixed partials" check to see if the following differential equation is exact.

If it is exact find a function  $F(x, y)$  whose differential,  $dF(x, y)$  is the left hand side of the differential equation. That is, level curves  $F(x, y) = C$  are solutions to the differential equation:

$$(4x^4 + 4y)dx + (1x - 1y^1)dy = 0$$

First:

$M_y(x, y) =$  \_\_\_\_\_, and  $N_x(x, y) =$  \_\_\_\_\_.

If the equation is not exact, enter *not exact*, otherwise enter in  $F(x, y)$  here \_\_\_\_\_.

**6. (1 pt) Library/Rochester/setDiffEQ3Separable/ns7.4.13a.pg**

Find an equation of the curve that satisfies

$$\frac{dy}{dx} = 78yx^{12}$$

and whose y-intercept is 6.

$y(x) =$  \_\_\_\_\_.

**7. (1 pt) Library/Rochester/setDiffEQ3Separable/ur.de.3.3.pg**

The differential equation

$$64 \frac{dy}{dx} = (25 - x^2)^{-1/2} \exp(-8y)$$

has an implicit general solution of the form  $F(x, y) = K$ .

In fact, because the differential equation is separable, we can define the solution curve implicitly by a function in the form

$$F(x, y) = G(x) + H(y) = K.$$

Find such a solution and then give the related functions requested.

$F(x, y) = G(x) + H(y) =$

\_\_\_\_\_.

**8. (1 pt) Library/Rochester/setDiffEQ3Separable/ns7.4.3a.pg**

Solve the separable differential equation

$$5yy' = x.$$

Use the following initial condition:  $y(5) = 10$ .

Express  $x^2$  in terms of  $y$ .

$x^2 =$  \_\_\_\_\_ (function of  $y$ ).

**9. (1 pt) Library/Rochester/setDiffEQ3Separable/ur.de.3.16.pg**

Solve the separable differential equation

$$\frac{dx}{dt} = x^2 + \frac{1}{16},$$

and find the particular solution satisfying the initial condition

$$x(0) = 6.$$

$x(t) =$  \_\_\_\_\_.

**10. (1 pt) Library/Rochester/setDiffEQ3Separable/ns7.4.8.pg**

Solve the differential equation

$$(y^{17}x) \frac{dy}{dx} = 1 + x.$$

Use the initial condition  $y(1) = 3$ .

Express  $y^{18}$  in terms of  $x$ .

$y^{18} =$  \_\_\_\_\_.

**11. (1 pt) Library/Rochester/setDiffEQ3Separable/ns7.4.10.pg**

Solve the separable differential equation

$$11x - 4y\sqrt{x^2 + 1} \frac{dy}{dx} = 0.$$

Subject to the initial condition:  $y(0) = 6$ .

$y =$  \_\_\_\_\_.

SOLUTION:

**SOLUTION:** (Instructor solution preview: show the student solution after due date. )

**Solution:**

First separate the variables:

$$11x = 4y\sqrt{x^2 + 1} \frac{dy}{dx}$$

$$\frac{11x}{\sqrt{x^2 + 1}} dx = 4y dy$$

Now integrate both sides:

$$\int \frac{11x}{\sqrt{x^2 + 1}} dx = \int 4y dy$$

On the left hand side, make the substitution  $u = x^2 + 1$ ,  $du = 2xdx$ ,  $\frac{1}{2}du = xdx$ , then

$$\int \frac{11x}{\sqrt{x^2 + 1}} dx = \int \frac{5.5}{\sqrt{u}} du = 5.5 \frac{\sqrt{u}}{1/2} + C$$

$$= 11\sqrt{u} + C = 11\sqrt{x^2 + 1} + C.$$

Thus we have

$$11\sqrt{x^2 + 1} + C = 2y^2$$

$$y^2 = \frac{11}{2}\sqrt{x^2 + 1} + \frac{C}{2}$$

$$y^2 = \frac{11}{2}\sqrt{x^2 + 1} + K.$$

$$y = \pm \sqrt{\frac{11}{2}\sqrt{x^2 + 1} + K}.$$

We need  $y(0) = 6$ , so we have to use +, and we find  $K$  from

$$6 = \sqrt{\frac{11}{2}\sqrt{0^2 + 1} + K}$$

$$36 = \frac{11}{2} + K$$

$$K = 36 - \frac{11}{2} = 30.5.$$

Therefore the solution is  $y = \sqrt{\frac{11}{2}\sqrt{x^2 + 1} + 30.5}$ .

**12. (1 pt) Library/Rochester/setDiffEQ3Separable/ur.de.3.15.pg**

Solve the separable differential equation

$$\frac{dy}{dx} = \frac{-0.4}{\cos(y)},$$

and find the particular solution satisfying the initial condition

$$y(0) = \frac{\pi}{2}.$$

$y(x) =$  \_\_\_\_\_.

**13. (1 pt) Library/Rochester/setDiffEQ3Separable/jas7.4.5b.pg**

Solve the separable differential equation for  $u$

$$\frac{du}{dt} = e^{5u+9t}.$$

Use the following initial condition:  $u(0) = -5$ .

$u =$  \_\_\_\_\_.

**14. (1 pt) Library/UMN/calculusStewartCCC/s.7.3\_prob01.pg**

Find the solution of the differential equation  $\frac{dy}{dx} = y^2 + 4$  that satisfies the initial condition  $y(2) = 0$ .

Answer:  $y(x) =$  \_\_\_\_\_

**15. (1 pt) Library/FortLewis/DiffEq/1-First-order/04-Linear-integrating-factor/KJ-2-2-37.pg**

(1) Find the solution to the initial value problem

$$\frac{y' - e^{-t} + 5}{y} = -5, \quad y(0) = -2.$$

\_\_\_\_\_

(2) Discuss the behavior of the solution  $y(t)$  as  $t$  becomes large. Does  $\lim_{t \rightarrow \infty} y(t)$  exist? If the limit exists, enter its value. If the limit does not exist, enter *DNE*.

$\lim_{t \rightarrow \infty} y(t) =$  \_\_\_\_\_

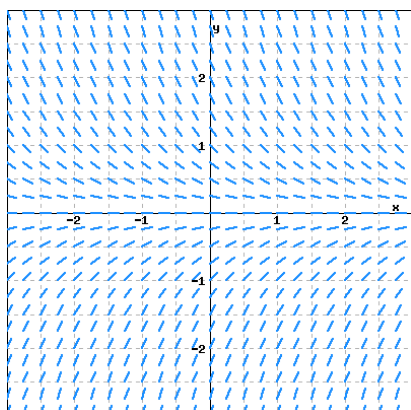
**16. (1 pt) Library/FortLewis/DiffEq/1-First-order/02-Slope-fields/KJ-1-3-14.pg**

Consider the slope field shown.

(a) For the solution that satisfies  $y(0) = 0$ , sketch the solution curve and estimate the following:  
 $y(1) \approx$  \_\_\_\_\_ and  $y(-1) \approx$  \_\_\_\_\_

(b) For the solution that satisfies  $y(0) = 1$ , sketch the solution curve and estimate the following:  
 $y(1) \approx$  \_\_\_\_\_ and  $y(-1) \approx$  \_\_\_\_\_

(c) For the solution that satisfies  $y(0) = -1$ , sketch the solution curve and estimate the following:  
 $y(1) \approx$  \_\_\_\_\_ and  $y(-1) \approx$  \_\_\_\_\_



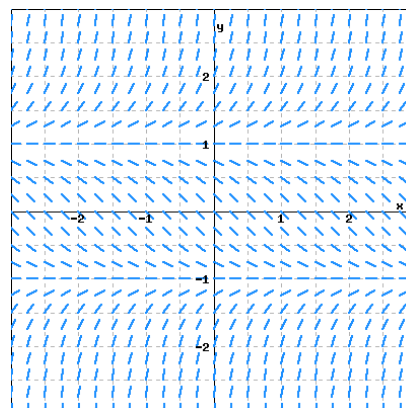
**17. (1 pt) Library/FortLewis/DiffEq/1-First-order/02-Slope-fields/KJ-1-3-16.pg**

Consider the slope field shown.

(a) For the solution that satisfies  $y(0) = 0$ , sketch the solution curve and estimate the following:  
 $y(1) \approx$  \_\_\_\_\_ and  $y(-1) \approx$  \_\_\_\_\_

(b) For the solution that satisfies  $y(0) = 1$ , sketch the solution curve and estimate the following:  
 $y(1) \approx$  \_\_\_\_\_ and  $y(-1) \approx$  \_\_\_\_\_

(c) For the solution that satisfies  $y(0) = -1$ , sketch the solution curve and estimate the following:  
 $y(1) \approx$  \_\_\_\_\_ and  $y(-1) \approx$  \_\_\_\_\_



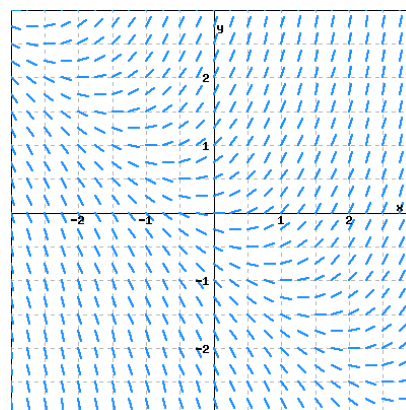
**18. (1 pt) Library/FortLewis/DiffEq/1-First-order/02-Slope-fields/KJ-1-3-18.pg**

Consider the slope field shown.

(a) For the solution that satisfies  $y(0) = 0$ , sketch the solution curve and estimate the following:  
 $y(1) \approx$  \_\_\_\_\_ and  $y(-1) \approx$  \_\_\_\_\_

(b) For the solution that satisfies  $y(0) = 1$ , sketch the solution curve and estimate the following:  
 $y(0.5) \approx$  \_\_\_\_\_ and  $y(-1) \approx$  \_\_\_\_\_

(c) For the solution that satisfies  $y(0) = -1$ , sketch the solution curve and estimate the following:  
 $y(1) \approx$  \_\_\_\_\_ and  $y(-1) \approx$  \_\_\_\_\_



**19. (1 pt) Library/FortLewis/DiffEq/1-First-order/08-Existence-uniqueness/KJ-2-1-06.pg**

Put the differential equation  $4ty + e^t y' = \frac{y}{t^2 + 16}$  into the form  $y' + p(t)y = g(t)$  and find  $p(t)$  and  $g(t)$ .

$p(t) =$  \_\_\_\_\_

$g(t) =$  \_\_\_\_\_

Is the differential equation  $4ty + e^t y' = \frac{y}{t^2 + 16}$  linear and homogeneous, linear and nonhomogeneous, or nonlinear?

Answer:

**20. (1 pt) Library/FortLewis/DiffEq/1-First-order/08-Existence-uniqueness/KJ-2-1-16.pg**

Consider the initial value problem

$$2ty' = 4y, \quad y(-2) = -4.$$

(1) Find the value of the constant  $C$  and the exponent  $r$  so that  $y = Ct^r$  is the solution of this initial value problem.  
 $y = \underline{\hspace{2cm}}$

(2) Determine the largest interval of the form  $a < t < b$  on which the existence and uniqueness theorem for first order linear differential equations guarantees the existence of a unique solution.  
 $\underline{\hspace{2cm}}$

(3) What is the actual interval of existence for the solution (from part a)?  
 $\underline{\hspace{2cm}}$

**21. (1 pt) Library/FortLewis/DiffEq/1-First-order/01-Integrals-as-solutions/Lebl-1-1-09.pg**

A spaceship is traveling directly away from earth with speed  $6t^2 + 1$  km/s. At time  $t = 0$  it is 1500 km from earth. How far is it from earth one minute after time  $t = 0$ ? Include units in your answer.

Distance =  $\underline{\hspace{2cm}}$

**22. (1 pt) Library/FortLewis/DiffEq/1-First-order/01-Integrals-as-solutions/Lebl-1-1-08.pg**

Solve  $y'' = \sin(x)$  if  $y(0) = 0$  and  $y'(0) = 8$ .

$y(x) = \underline{\hspace{2cm}}$

**23. (1 pt) Library/FortLewis/DiffEq/1-First-order/03-Separable/Lebl-1-3-01.pg**

Solve the differential equation  $\frac{dy}{dx} = \frac{x}{4y}$ .

(1) Find an implicit solution and put your answer in the following form:  
 $\underline{\hspace{2cm}} = \text{constant}.$

(2) Find the equation of the solution through the point  $(x, y) = (-2, 1)$ .  
 $\underline{\hspace{2cm}}$

(3) Find the equation of the solution through the point  $(x, y) = (0, -4)$ . Your answer should be of the form  $y = f(x)$ .  
 $\underline{\hspace{2cm}}$

**24. (1 pt) Library/maCalcDB/setDiffEQ3Separable/ns7.4.10.pg**

Solve the separable differential equation

$$10x - 3y\sqrt{x^2 + 1} \frac{dy}{dx} = 0.$$

Subject to the initial condition:  $y(0) = 10$ .

$y = \underline{\hspace{2cm}}$ .

**25. (1 pt) Library/Michigan/Chap11Sec4/Q43.pg**

Find the general solution to the differential equation modeling how a person learns:

$$\frac{dy}{dt} = 100 - y.$$

Then find the particular solutions with the following initial conditions:

$y(0) = 5$  :  $y = \underline{\hspace{2cm}}$

$y(0) = 125$  :  $y = \underline{\hspace{2cm}}$

Plot the slope field of this differential equation and sketch the solutions with  $y(0) = 5$  and  $y(0) = 125$ .

Which of these two particular solutions could represent how a person learns?

- A.  $y(0) = 5$
- B.  $y(0) = 125$
- C. either of these
- D. none of the above

SOLUTION:

**SOLUTION:** (Instructor solution preview: show the student solution after due date. )

SOLUTION

Separating variables and integrating gives

$$\int \frac{1}{100 - y} dy = \int dt,$$

so that

$$-\ln|100 - y| = t + C$$

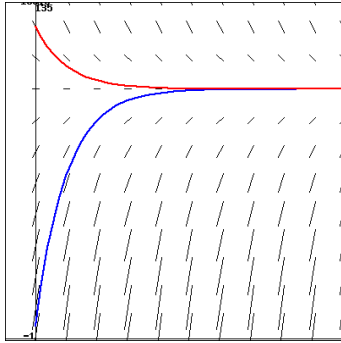
or

$$y(t) = 100 - Ae^{-t}.$$

Thus with the initial condition  $y(0) = 5$  we must have  $5 = 100 - A$ , or  $A = 100 - 5 = 95$ . Similarly, if  $y(0) = 125$ ,  $A = 100 - 125 = -$ . Thus the two particular solutions are

$$y = 100 - 95e^{-t} \quad \text{and} \quad y = 100 + 25e^{-t}.$$

The slope field and graphs of the two solutions (that with  $y(0) = 5$  in blue and the other in red) is shown below.



*(Click on the graph for a larger version.)*

Because the second solution would require a person to start off knowing more than 100 percent of the information, the first is that which could represent how a person learns.