1. **(1 pt) Library/FortLewis/DiffEq/0-Introduction/KJ-1-2-20a.pg**
Given that $y(t) = c_1 e^{4t} + c_2 e^{-4t}$ is a solution to the differential equation $y'' - 16y = 0$, where c_1 and c_2 are arbitrary constants, find a function $y(t)$ that satisfies the conditions

- $y'' - 16y = 0,$
- $y(0) = 5,$
- $\lim_{t\to-\infty} y(t) = 0.$

$y(t) =$ ____________

2. **(1 pt) Library/FortLewis/DiffEq/0-Introduction/KJ-1-2-20a.pg**
Given that $y(t) = c_1 e^{4t} + c_2 e^{-4t}$ is a solution to the differential equation $y'' - 16y = 0$, where c_1 and c_2 are arbitrary constants, find a function $y(t)$ that satisfies the conditions

- $y'' - 16y = 0,$
- $y(0) = 5,$
- $\lim_{t\to-\infty} y(t) = 0.$

$y(t) =$ ____________

3. **(1 pt) Library/FortLewis/DiffEq/0-Introduction/KJ-1-2-20a.pg**
Given that $y(t) = c_1 e^{4t} + c_2 e^{-4t}$ is a solution to the differential equation $y'' - 16y = 0$, where c_1 and c_2 are arbitrary constants, find a function $y(t)$ that satisfies the conditions

- $y'' - 16y = 0,$
- $y(0) = 5,$
- $\lim_{t\to-\infty} y(t) = 0.$

$y(t) =$ ____________

4. **(1 pt) Library/FortLewis/DiffEq/2-Higher-order/02-Linear-2nd-order-cc/Lebl-2-2-08.pg**
(a) Find the general solution to $y'' - 12y' + 36y = 0$. In your answer, use c_1 and c_2 to denote arbitrary constants and x the independent variable. Enter c_1 as c1 and c_2 as c2.

____

(b) Find the solution that satisfies the initial conditions $y(0) = 4$ and $y'(0) = 0$.

____

5. **(1 pt) Library/FortLewis/DiffEq/2-Higher-order/02-Linear-2nd-order-cc/Lebl-2-2-08.pg**
(a) Find the general solution to $y'' - 10y' + 25y = 0$. In your answer, use c_1 and c_2 to denote arbitrary constants and x the independent variable. Enter c_1 as c1 and c_2 as c2.

____

(b) Find the solution that satisfies the initial conditions $y(0) = 3$ and $y'(0) = 0$.

____

6. **(1 pt) Library/FortLewis/DiffEq/2-Higher-order/02-Linear-2nd-order-cc/Lebl-2-2-08.pg**
(a) Find the general solution to $y'' - 12y' + 36y = 0$. In your answer, use c_1 and c_2 to denote arbitrary constants and x the independent variable. Enter c_1 as c1 and c_2 as c2.

____

(b) Find the solution that satisfies the initial conditions $y(0) = 4$ and $y'(0) = 0$.

____

7. **(1 pt) Library/FortLewis/DiffEq/2-Higher-order/02-Linear-2nd-order-cc/Lebl-2-2-11.pg**
Find the general solution to $y'' + 10y' + 29y = 0$. Give your answer as $y = ...$ In your answer, use c_1 and c_2 to denote arbitrary constants and x the independent variable. Enter c_1 as c1 and c_2 as c2.

____

8. **(1 pt) Library/FortLewis/DiffEq/2-Higher-order/02-Linear-2nd-order-cc/Lebl-2-2-11.pg**
Find the general solution to $y'' + 8y' + 25y = 0$. Give your answer as $y = ...$ In your answer, use c_1 and c_2 to denote arbitrary constants and x the independent variable. Enter c_1 as c1 and c_2 as c2.

____

9. **(1 pt) Library/FortLewis/DiffEq/2-Higher-order/02-Linear-2nd-order-cc/Lebl-2-2-09.pg**
(a) Find the general solution to $y'' + 7y' = 0$. Give your answer as $y = \ldots$. In your answer, use c_1 and c_2 to denote arbitrary constants and x the independent variable. Enter c_1 as c1 and c_2 as c2.

____

(b) Find the particular solution that satisfies $y(0) = 1$ and

$y'(0) = 1$.

______

10. (1 pt) Library/FortLewis/DiffEq/2-Higher-order/02-Linear-2nd-order-cc/Lebl-2-2-09.pg

(a) Find the general solution to $y'' + 2y' = 0$. Give your answer as $y = \ldots$. In your answer, use c_1 and c_2 to denote arbitrary constants and x the independent variable. Enter c_1 as c1 and c_2 as c2.

______

(b) Find the particular solution that satisfies $y(0) = 1$ and $y'(0) = 1$.

______

11. (1 pt) Library/FortLewis/DiffEq/2-Higher-order/02-Linear-2nd-order-cc/Lebl-2-2-09.pg

(a) Find the general solution to $y'' + 6y' = 0$. Give your answer as $y = \ldots$. In your answer, use c_1 and c_2 to denote arbitrary constants and x the independent variable. Enter c_1 as c1 and c_2 as c2.

______

(b) Find the particular solution that satisfies $y(0) = 1$ and $y'(0) = 1$.

______

12. (1 pt) Library/FortLewis/DiffEq/2-Higher-order/02-Linear-2nd-order-cc/Lebl-2-2-06.pg

Find the general solution to $5y'' + 10y' - 15y = 0$. In your answer, use c_1 and c_2 to denote arbitrary constants and x the independent variable. Enter c_1 as c1 and c_2 as c2.

______

13. (1 pt) Library/FortLewis/DiffEq/2-Higher-order/02-Linear-2nd-order-cc/Lebl-2-2-06.pg

Find the general solution to $7y'' + 7y' - 14y = 0$. In your answer, use c_1 and c_2 to denote arbitrary constants and x the independent variable. Enter c_1 as c1 and c_2 as c2.

______

14. (1 pt) Library/FortLewis/DiffEq/2-Higher-order/04-Mechanical-vibrations/Lebl-2-4-03.pg

Suppose a spring with spring constant 3 $\mathrm{N/m}$ is horizontal and has one end attached to a wall and the other end attached to a 3 kg mass. Suppose that the friction of the mass with the floor (i.e., the damping constant) is 6 $\mathrm{N \cdot s/m}$.

(1) Set up a differential equation that describes this system. Let x to denote the displacement, in meters, of the mass from its equilibrium position, and give your answer in terms of x, x', x''. Assume that positive displacement means the mass is farther from the wall than when the system is at equilibrium.

______

(2) Find the general solution to your differential equation from the previous part. Use c_1 and c_2 to denote arbitrary constants. Use t for independent variable to represent the time elapsed in seconds. Enter c_1 as c1 and c_2 as c2.

______

(3) Is this system under damped, over damped, or critically damped? [?] Enter a value for the damping constant that would make the system critically damped.
_____ $\mathrm{N \cdot s/m}$

______

15. (1 pt) Library/FortLewis/DiffEq/2-Higher-order/04-Mechanical-vibrations/Lebl-2-4-03.pg

Suppose a spring with spring constant 4 $\mathrm{N/m}$ is horizontal and has one end attached to a wall and the other end attached to a 4 kg mass. Suppose that the friction of the mass with the floor (i.e., the damping constant) is 8 $\mathrm{N \cdot s/m}$.

(1) Set up a differential equation that describes this system. Let x to denote the displacement, in meters, of the mass from its equilibrium position, and give your answer in terms of x, x', x''. Assume that positive displacement means the mass is farther from the wall than when the system is at equilibrium.

______

(2) Find the general solution to your differential equation from the previous part. Use c_1 and c_2 to denote arbitrary constants. Use t for independent variable to represent the time elapsed in seconds. Enter c_1 as c1 and c_2 as c2.

______

(3) Is this system under damped, over damped, or critically damped? [?] Enter a value for the damping constant that would make the system critically damped.
_____ $\mathrm{N \cdot s/m}$

______

16. (1 pt) Library/FortLewis/DiffEq/2-Higher-order/04-Mechanical-vibrations/KJ-3-6-08.pg

The graph shows the displacement from equilibrium of a mass-spring system as a function of time after the vertically hanging system was set in motion at time $t = 0$. Assume that the units of time are seconds, and the units of displacement are centimeters. The first t-intercept is $(0.75, 0)$ and the first maximum has coordinates $(1.25, 2)$.

(a) What is the period T of the periodic motion?
$T =$ ______ seconds

(b) What is the frequency f in Hertz? What is the angular frequency ω in radians / second?
$f =$ ______ Hertz
$\omega =$ ______ radians / second

(d) Determine the amplitude A and the phase angle γ (in radians), and express the displacement in the form $y(t) = A\cos(\omega t - \gamma)$, with y in meters.
$y(t) =$ ____________ meters

(e) With what initial displacement $y(0)$ and initial velocity $y'(0)$ was the system set into motion?
$y(0) =$ ____ meters
$y'(0) =$ ____ meters / second

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The graph shows the displacement from equilibrium of a mass-spring system as a function of time after the vertically hanging system was set in motion at time $t = 0$. Assume that the units of time are seconds, and the units of displacement are centimeters. The first t-intercept is $(0.25, 0)$ and the first minimum has coordinates $(1.25, -3)$.

(a) What is the period T of the periodic motion?
$T =$ ______ seconds

(b) What is the frequency f in Hertz? What is the angular frequency ω in radians / second?
$f =$ ______ Hertz
$\omega =$ ______ radians / second

(d) Determine the amplitude A and the phase angle γ (in radians), and express the displacement in the form $y(t) = A\cos(\omega t - \gamma)$, with y in meters.
$y(t) =$ ____________ meters

(e) With what initial displacement $y(0)$ and initial velocity $y'(0)$ was the system set into motion?
$y(0) =$ ____ meters
$y'(0) =$ ____ meters / second

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17. **(1 pt) Library/FortLewis/DiffEq/2-Higher-order/04-Mechanical-vibrations/KJ-3-6-08.pg**

18. **(1 pt) Library/FortLewis/DiffEq/2-Higher-order/04-Mechanical-vibrations/KJ-3-6-08.pg**

The graph shows the displacement from equilibrium of a mass-spring system as a function of time after the vertically hanging system was set in motion at time $t = 0$. Assume that the units of time are seconds, and the units of displacement are centimeters. The first t-intercept is $(0.25, 0)$ and the first minimum has coordinates $(1.25, -3)$.

(a) What is the period T of the periodic motion?
$T =$ _______ seconds

(b) What is the frequency f in Hertz? What is the angular frequency ω in radians / second?
$f =$ _______ Hertz
$\omega =$ _______ radians / second

(d) Determine the amplitude A and the phase angle γ (in radians), and express the displacement in the form $y(t) = A\cos(\omega t - \gamma)$, with y in meters.
$y(t) =$ ______________ meters

(e) With what initial displacement $y(0)$ and initial velocity $y'(0)$ was the system set into motion?
$y(0) =$ _____ meters
$y'(0) =$ _____ meters / second

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19. **(1 pt) Library/FortLewis/DiffEq/2-Higher-order/04-Mechanical-vibrations/KJ-3-6-08.pg**

The graph shows the displacement from equilibrium of a mass-spring system as a function of time after the vertically hanging system was set in motion at time $t = 0$. Assume that the units of time are seconds, and the units of displacement are centimeters. The first t-intercept is $(0.75, 0)$ and the first maximum has coordinates $(1.25, 4)$.

(a) What is the period T of the periodic motion?
$T =$ _______ seconds

(b) What is the frequency f in Hertz? What is the angular frequency ω in radians / second?
$f =$ _______ Hertz
$\omega =$ _______ radians / second

(d) Determine the amplitude A and the phase angle γ (in radians), and express the displacement in the form $y(t) = A\cos(\omega t - \gamma)$, with y in meters.
$y(t) =$ ______________ meters

(e) With what initial displacement $y(0)$ and initial velocity $y'(0)$ was the system set into motion?
$y(0) =$ _____ meters
$y'(0) =$ _____ meters / second

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20. **(1 pt) Library/FortLewis/DiffEq/2-Higher-order/04-Mechanical-vibrations/Lebl-2-4-03.pg**

Suppose a spring with spring constant $16\ \mathrm{N/m}$ is horizontal and has one end attached to a wall and the other end attached to a 4 kg mass. Suppose that the friction of the mass with the floor (i.e., the damping constant) is $16\ \mathrm{N \cdot s/m}$.

(1) Set up a differential equation that describes this system. Let x to denote the displacement, in meters, of the mass

from its equilibrium position, and give your answer in terms of x, x', x''. Assume that positive displacement means the mass is farther from the wall than when the system is at equilibrium.

____

(2) Find the general solution to your differential equation from the previous part. Use c_1 and c_2 to denote arbitrary constants. Use t for independent variable to represent the time elapsed in seconds. Enter c_1 as c1 and c_2 as c2.

____

(3) Is this system under damped, over damped, or critically damped? [?] Enter a value for the damping constant that would make the system critically damped.
_____ $\text{N}\cdot\text{s/m}$

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21. **(1 pt) Library/FortLewis/DiffEq/2-Higher-order/03-Linear-higher-order/Lebl-2-3-10.pg**
Are the functions f, g, and h given below linearly independent?

$$f(x) = e^{3x}, \quad g(x) = xe^{3x}, \quad h(x) = x^2 e^{3x}.$$

If they are independent, enter all zeroes. If they are not linearly independent, find a nontrivial solution to the equation below. Be sure you can justify your answer.

$$\_\_\_ \left(e^{3x}\right) + \_\_\_ \left(xe^{3x}\right) + \_\_\_ \left(x^2 e^{3x}\right) = 0.$$

---

22. **(1 pt) Library/FortLewis/DiffEq/2-Higher-order/03-Linear-higher-order/Lebl-2-3-10.pg**
Are the functions f, g, and h given below linearly independent?

$$f(x) = e^{3x}, \quad g(x) = xe^{3x}, \quad h(x) = x^2 e^{3x}.$$

If they are independent, enter all zeroes. If they are not linearly independent, find a nontrivial solution to the equation below. Be sure you can justify your answer.

$$\_\_\_ \left(e^{3x}\right) + \_\_\_ \left(xe^{3x}\right) + \_\_\_ \left(x^2 e^{3x}\right) = 0.$$

---

23. **(1 pt) Library/FortLewis/DiffEq/2-Higher-order/03-Linear-higher-order/Lebl-2-3-10.pg**
Are the functions f, g, and h given below linearly independent?

$$f(x) = e^{3x}, \quad g(x) = xe^{3x}, \quad h(x) = x^2 e^{3x}.$$

If they are independent, enter all zeroes. If they are not linearly independent, find a nontrivial solution to the equation below. Be sure you can justify your answer.

$$\_\_\_ \left(e^{3x}\right) + \_\_\_ \left(xe^{3x}\right) + \_\_\_ \left(x^2 e^{3x}\right) = 0.$$

---

24. **(1 pt) Library/FortLewis/DiffEq/2-Higher-order/03-Linear-higher-order/Lebl-2-3-08.pg**
Are the functions f, g, and h given below linearly independent?

$$f(x) = 0, \quad g(x) = \cos(5x), \quad h(x) = \sin(5x).$$

If they are independent, enter all zeroes. If they are not linearly independent, find a nontrivial solution to the equation below. Be sure you can justify your answer.

$$\_\_\_ (0) + \_\_\_ (\cos(5x)) + \_\_\_ (\sin(5x)) = 0.$$

---

25. **(1 pt) Library/FortLewis/DiffEq/2-Higher-order/03-Linear-higher-order/Lebl-2-3-02.pg**
Find the general solution to $y^{(4)} - 7y''' + 12y'' = 0$. In your answer, use c_1, c_2, c_3 and c_4 to denote arbitrary constants and x the independent variable. Enter c_1 as c1, c_2 as c2, etc.

____

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26. **(1 pt) Library/FortLewis/DiffEq/2-Higher-order/03-Linear-higher-order/Lebl-2-3-01.pg**
Find the general solution to $y''' - y'' + 2y' - 2y = 0$. In your answer, use c_1, c_2 and c_3 to denote arbitrary constants and x the independent variable. Enter c_1 as c1, c_2 as c2, and c_3 as c3.

____

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27. **(1 pt) Library/FortLewis/DiffEq/2-Higher-order/03-Linear-higher-order/Lebl-2-3-06.pg**
Suppose that a fourth order differential equation has a solution $y = -9e^{3x}x\cos(x)$.

(a) Find such a differential equation, assuming it is homogeneous and has constant coefficients.

____

(b) Find the general solution to this differential equation. In your answer, use c_1, c_2, c_3 and c_4 to denote arbitrary constants and x the independent variable. Enter c_1 as c1, c_2 as c2, etc.

____

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28. **(1 pt) Library/FortLewis/DiffEq/2-Higher-order/03-Linear-higher-order/Lebl-2-3-07.pg**
Are the functions f, g, and h given below linearly independent?

$$f(x) = e^{3x} - \cos(6x), \quad g(x) = e^{3x} + \cos(6x), \quad h(x) = \cos(6x).$$

If they are independent, enter all zeroes. If they are not linearly independent, find a nontrivial solution to the equation below. Be sure you can justify your answer.

$$\_\_\_ \left(e^{3x} - \cos(6x)\right) + \_\_\_ \left(e^{3x} + \cos(6x)\right) + \_\_\_ (\cos(6x)) = 0.$$

29. **(1 pt) Library/FortLewis/DiffEq/2-Higher-order/03-Linear-higher-order/Lebl-2-3-04.pg**
Suppose that the characteristic equation for a differential equation is $(r-2)^2(r-5)^2 = 0$.

(a) Find such a differential equation, assuming it is homogeneous and has constant coefficients. Enter your answer using y, y', y'', y''', y'''' for the dependent variable and its derivatives.

____

(b) Find the general solution to this differential equation. In your answer, use c_1, c_2, c_3 and c_4 to denote arbitrary constants, use y for the dependent variable, and use x for the independent variable. Enter c_1 as c1, c_2 as c2, etc.

____

30. **(1 pt) Library/FortLewis/DiffEq/2-Higher-order/05-Nonhomogeneous/Lebl-2-5-02.pg**

(1) Find a particular solution to the nonhomogeneous differential equation $y'' + 3y' - 10y = e^{6x}$.

$y_p =$ ____________________

(2) Find the most general solution to the associated homogeneous differential equation. Use c_1 and c_2 in your answer to denote arbitrary constants, and enter them as c1 and c2.

$y_h =$ ____________________

(3) Find the most general solution to the original nonhomogeneous differential equation. Use c_1 and c_2 in your answer to denote arbitrary constants.

$y =$ ____________________

31. **(1 pt) Library/FortLewis/DiffEq/2-Higher-order/05-Nonhomogeneous/Lebl-2-5-03.pg**

(1) Find a particular solution to the nonhomogeneous differential equation $y'' - 6y' + 9y = e^{3x}$.

$y_p =$ ____________________

(2) Find the most general solution to the associated homogeneous differential equation. Use c_1 and c_2 in your answer to denote arbitrary constants and enter them as c1 and c2.

$y_h =$ ____________________

(3) Find the most general solution to the original nonhomogeneous differential equation. Use c_1 and c_2 in your answer to denote arbitrary constants.

$y =$ ____________________

32. **(1 pt) Library/FortLewis/DiffEq/2-Higher-order/05-Nonhomogeneous/KJ-3-8-28.pg**
Consider the differential equation

$$y'' + \alpha y' + \beta y = t + e^{4t}.$$

Suppose the form of the particular solution to this differential equation as prescribed by the method of undetermined coefficients is

$$y_p(t) = A_1 t^2 + A_0 t + B_0 t e^{4t}.$$

Determine the constants α and β.

$\alpha =$ _____

$\beta =$ _____

33. **(1 pt) Library/FortLewis/DiffEq/2-Higher-order/05-Nonhomogeneous/KJ-3-8-28.pg**
Consider the differential equation

$$y'' + \alpha y' + \beta y = t + e^{6t}.$$

Suppose the form of the particular solution to this differential equation as prescribed by the method of undetermined coefficients is

$$y_p(t) = A_1 t^2 + A_0 t + B_0 t e^{6t}.$$

Determine the constants α and β.

$\alpha =$ _____

$\beta =$ _____

34. **(1 pt) Library/FortLewis/DiffEq/2-Higher-order/05-Nonhomogeneous/KJ-3-8-14.pg**

(1) Find a particular solution to the nonhomogeneous differential equation $y'' + 4y' + 5y = 5x + 5e^{-x}$.

$y_p =$ ______________________________

(2) Find the most general solution to the associated homogeneous differential equation. Use c_1 and c_2 in your answer to denote arbitrary constants, and enter them as c1 and c2.

$y_h =$ ______________________________

(3) Find the most general solution to the original nonhomogeneous differential equation. Use c_1 and c_2 in your answer to denote arbitrary constants.

$y =$ ______________________________

35. **(1 pt) Library/FortLewis/DiffEq/2-Higher-order/05-Nonhomogeneous/Lebl-2-5-04.pg**

(1) Find a particular solution to the nonhomogeneous differential equation $y'' + 9y = \cos(3x) + \sin(3x)$.

$y_p =$ ______________________________

(2) Find the most general solution to the associated homogeneous differential equation. Use c_1 and c_2 in your answer to denote arbitrary constants. Enter c_1 as c1 and c_2 as c2.

$y_h =$ ______________________________

(3) Find the solution to the original nonhomogeneous differential equation satisfying the initial conditions $y(0) = 6$ and $y'(0) = 2$.

$y =$ ______________________________

36. **(1 pt) Library/FortLewis/DiffEq/2-Higher-order/05-Nonhomogeneous/KJ-3-8-31.pg**

Consider the initial value problem

$$y'' + 4y = e^{-t}, \quad y(0) = y_0, \quad y'(0) = y_0'.$$

Suppose we know that $y(t) \to 0$ as $t \to \infty$. Determine the solution and the initial conditions.

$y(t) =$ ______________

$y(0) =$ _____

$y'(0) =$ _____

37. **(1 pt) Library/FortLewis/DiffEq/2-Higher-order/06-Forcing-resonance/KJ-3-10-07.pg**

Consider the initial value problem

$$my'' + cy' + ky = F(t), \quad y(0) = 0, \quad y'(0) = 0$$

modeling the motion of a spring-mass-dashpot system initially at rest and subjected to an applied force $F(t)$, where the unit of force is the Newton (N). Assume that $m = 2$ kilograms, $c = 8$ kilograms per second, $k = 80$ Newtons per meter, and $F(t) = 80\cos(8t)$ Newtons.

(1) Solve the initial value problem.

$y(t) =$

____

(2) Determine the long-term behavior of the system. Is $\lim_{t\to\infty} y(t) = 0$? If it is, enter zero. If not, enter a function that approximates $y(t)$ for very large positive values of t.

For very large positive values of t, $y(t) \approx$ ______________________

38. **(1 pt) Library/FortLewis/DiffEq/2-Higher-order/06-Forcing-resonance/KJ-3-10-02.pg**

A 10 kilogram object suspended from the end of a vertically hanging spring stretches the spring 9.8 centimeters. At time $t = 0$, the resulting mass-spring system is disturbed from its rest state by the force $F(t) = 130\cos(10t)$. The force $F(t)$ is expressed in Newtons and is positive in the downward direction, and time is measured in seconds.

(a) Determine the spring constant k.
$k =$ _____ Newtons / meter

(b) Formulate the initial value problem for $y(t)$, where $y(t)$ is the displacement of the object from its equilibrium rest state, measured positive in the downward direction. (Give your answer in terms of y, y', y'', t.

Differential equation: ____________________

Initial conditions: $y(0) =$ ___ and $y'(0) =$ ___

(c) Solve the initial value problem for $y(t)$.
$y(t) =$ ____________________

(d) Plot the solution and determine the maximum excursion from equilibrium made by the object on the time interval $0 \le t < \infty$. If there is no such maximum, enter *NONE* .
maximum excursion = __________ meters

39. **(1 pt) Library/FortLewis/DiffEq/2-Higher-order/06-Forcing-resonance/KJ-3-10-08.pg**

Consider the initial value problem

$$my'' + cy' + ky = F(t), \quad y(0) = 0, \quad y'(0) = 0$$

modeling the motion of a spring-mass-dashpot system initially at rest and subjected to an applied force $F(t)$, where the unit of force is the Newton (N). Assume that $m = 2$ kilograms, $c = 8$ kilograms per second, $k = 80$ Newtons per meter, and $F(t) = 50e^{-t}$ Newtons.

(a) Solve the initial value problem.

$y(t) =$ ______________________

(b) Determine the long-term behavior of the system. Is $\lim_{t\to\infty} y(t) = 0$? If it is, enter zero. If not, enter a function that approximates $y(t)$ for very large positive values of t.

For very large positive values of t, $y(t) \approx$ ______________________

40. **(1 pt) Library/FortLewis/DiffEq/2-Higher-order/06-Forcing-resonance/KJ-3-10-10.pg**

Consider the initial value problem

$$my'' + cy' + ky = F(t), \quad y(0) = 0, \quad y'(0) = 0$$

modeling the motion of a spring-mass-dashpot system initially at rest and subjected to an applied force $F(t)$, where the unit of force is the Newton (N). Assume that $m = 2$ kilograms, $c = 8$ kilograms per second, $k = 80$ Newtons per meter, and the applied force in Newtons is

$$F(t) = \begin{cases} 30 & \text{if } 0 \le t \le \pi/2, \\ 0 & \text{if } t > \pi/2. \end{cases}$$

(i) Solve the initial value problem, using that the displacement $y(t)$ and velocity $y'(t)$ remain continuous when the applied force is discontinuous.

For $0 \le t \le \pi/2$, $y(t) =$ ______________________

For $t > \pi/2$, $y(t) =$ ______________________

(ii) Determine the long-term behavior of the system. Is $\lim_{t\to\infty} y(t) = 0$? If it is, enter zero. If not, enter a function that approximates $y(t)$ for very large positive values of t.

For very large positive values of t, $y(t) \approx$ ______________________