1. **(1 pt) Library/Rochester/setLinearAlgebra13ComplexEigenvalues-/ur_la_13_1.pg**

The matrix
$A = \begin{bmatrix} -3 & 7 \\ -7 & -5 \end{bmatrix}$
has complex eigenvalues, $\lambda_{1,2} = a \pm bi$, where $a =$ _____ and $b =$ _____.

2. **(1 pt) Library/Rochester/setLinearAlgebra13ComplexEigenvalues-/ur_la_13_2.pg**

Find all the eigenvalues (real and complex) of the matrix
$M = \begin{bmatrix} 2 & -4 & 0 \\ 3 & -2 & 0 \\ -3 & 0 & 2 \end{bmatrix}$.

Enter your answers in the following blank, separated by commas:

____

3. **(1 pt) Library/Rochester/setLinearAlgebra11Eigenvalues-/ur_la_11_1.pg**

Find the characteristic polynomial of the matrix
$A = \begin{bmatrix} 8 & -3 \\ 2 & -6 \end{bmatrix}$.
$p(x) =$ __________.

4. **(1 pt) Library/Rochester/setLinearAlgebra11Eigenvalues-/ur_la_11_20.pg**

The matrix $A = \begin{bmatrix} -2 & 0 & 4 \\ 0 & 0 & 0 \\ -1 & 0 & 2 \end{bmatrix}$
has one real eigenvalue. Find this eigenvalue, its multiplicity, and the dimension of the corresponding eigenspace.
eigenvalue = _____,
multiplicity = _____,
dimension of the eigenspace = _____.

5. **(1 pt) Library/Rochester/setLinearAlgebra11Eigenvalues-/ur_la_11_17.pg**

The matrix $A = \begin{bmatrix} -1 & 1 \\ -4 & 3 \end{bmatrix}$
has one eigenvalue of multiplicity 2. Find this eigenvalue and the dimenstion of the eigenspace.
eigenvalue = _____,
dimension of the eigenspace = _____.

6. **(1 pt) Library/TCNJ/TCNJ_Eigenvalues/problem7.pg**

Determine if v is an eigenvector of the matrix A.

[?] 1. $A = \begin{bmatrix} 8.88178419700125e-16 & 3 \\ -2 & 5 \end{bmatrix}$, $v = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

[?] 2. $A = \begin{bmatrix} -19 & -11 \\ 22 & 14 \end{bmatrix}$, $v = \begin{bmatrix} 6 \\ -3 \end{bmatrix}$

[?] 3. $A = \begin{bmatrix} -52 & -22 \\ 110 & 47 \end{bmatrix}$, $v = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

7. **(1 pt) Library/TCNJ/TCNJ_LinearAlgebra_DiffEquations-/problem1.pg**

Let $x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ be a solution to the system of differential equations:

$$\begin{aligned} x_1'(t) &= -10x_1(t) - 2x_2(t) \\ x_2'(t) &= 3x_1(t) - 5x_2(t) \end{aligned}$$

If $x(0) = \begin{bmatrix} -3 \\ -3 \end{bmatrix}$, find $x(t)$. Put the eigenvalues in ascending order when you enter $x_1(t), x_2(t)$ below.

$x_1(t) =$ ___ $exp($ ___ $t)+$ ___ $exp($ ___ $t)$

$x_2(t) =$ ___ $exp($ ___ $t)+$ ___ $exp($ ___ $t)$

8. **(1 pt) Library/FortLewis/DiffEq/3-Linear-systems/03-Linear-systems/KJ-4-3-05-a-multians.pg**

This is the first part of a two-part problem.

Rewrite the given system of linear homogeneous differential equations as a homogeneous linear system of the form $\vec{y}' = P\vec{y}$.

$$\begin{aligned} y_1' &= 2y_1 + y_2 + y_3, \\ y_2' &= y_2 + 2y_3 + y_1, \\ y_3' &= 2y_2 + y_1 + y_3. \end{aligned}$$

$$\begin{bmatrix} y_1' \\ y_2' \\ y_3' \end{bmatrix} = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

9. **(1 pt) Library/FortLewis/DiffEq/3-Linear-systems/03-Linear-systems/KJ-4-3-05-b-multians.pg**

This is the second part of a two-part problem.

Show that

$$\vec{y}(t) = \begin{bmatrix} e^{4t} \\ e^{4t} \\ e^{4t} \end{bmatrix}$$

is a solution to the system of linear homogeneous differential equations

$$\begin{aligned} y_1' &= 2y_1 + y_2 + y_3, \\ y_2' &= y_1 + y_2 + 2y_3, \\ y_3' &= y_1 + 2y_2 + y_3. \end{aligned}$$

(1) Find the value of each term in the equation $y_1' = 2y_1 + y_2 + y_3$ in terms of the variable t. (Enter the terms in the order given.)

_____ = _____ + _____ + _____.

(2) Find the value of each term in the equation $y_2' = y_1 + y_2 + 2y_3$ in terms of the variable t. (Enter the terms in the order given.)

_____ = _____ + _____ + _____.

(3) Find the value of each term in the equation $y_3' = y_1 + 2y_2 + y_3$ in terms of the variable t. (Enter the terms in the order given.)

_____ = _____ + _____ + _____.

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10. **(1 pt) Library/FortLewis/DiffEq/3-Linear-systems/03-Linear-systems/KJ-4-2-15.pg**

Consider the initial value problem

$$\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3 & -6 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2\cos(3t) \end{bmatrix},$$

$$\begin{bmatrix} y_1(3) \\ y_2(3) \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \end{bmatrix}.$$

This initial value problem was obtained from an initial value problem for a higher order scalar differential equation, via the change of variables $y_1 = y$ and $y_2 = y'$. What is the corresponding scalar initial value problem?

(1) Differential equation: ______________________

(Give your answer in terms of y, y', y'', t.)

(2) Initial conditions: _______ and _______

(Give your first answer in the form $y(t_0) = y_0$.
Give your second answer in the form $y'(t_0) = y_0'$.)

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11. **(1 pt) Library/FortLewis/DiffEq/3-Linear-systems/03-Linear-systems/Lebl-3-3-01.pg**

Write the system $x' = e^{3t}x - 8ty + 4\sin(t),\ y' = 4\tan(t)y + 4x - 3\cos(t)$ in the form

$$\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = P(t)\begin{bmatrix} x \\ y \end{bmatrix} + \vec{f}(t).$$

Use prime notation for derivatives and write x and x', etc., instead of $x(t)$, $x'(t)$, or $\frac{dx}{dt}$.

$$\begin{bmatrix} \_\_\_ \\ \_\_\_ \end{bmatrix} = \begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix} \begin{bmatrix} \_\_\_ \\ \_\_\_ \end{bmatrix} + \begin{bmatrix} \_\_\_ \\ \_\_\_ \end{bmatrix}$$

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12. **(1 pt) Library/FortLewis/DiffEq/3-Linear-systems/03-Linear-systems/KJ-4-3-16-b-multians.pg**

This is the second part of a four-part problem.

Let

$$\vec{y}_1(t) = \begin{bmatrix} 2e^{3t} - 8e^{-t} \\ 3e^{3t} - 20e^{-t} \end{bmatrix}, \quad \vec{y}_2(t) = \begin{bmatrix} -6e^{3t} + 2e^{-t} \\ -9e^{3t} + 5e^{-t} \end{bmatrix}.$$

Compute the Wronskian to determine whether the functions $\vec{y}_1(t)$ and $\vec{y}_2(t)$ are linearly independent.

$$\text{Wronskian} = \det\begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix} = \_\_\_\_\_\_$$

These functions are linearly [?] because the Wronskian is [?] for all t. Therefore, the solutions $\vec{y}_1(t)$ and $\vec{y}_2(t)$ to the system

$$\vec{y}' = \begin{bmatrix} 9 & -4 \\ 15 & -7 \end{bmatrix}\vec{y}$$

[?] form a fundamental set (i.e., linearly independent set) of solutions.

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13. **(1 pt) Library/FortLewis/DiffEq/3-Linear-systems/03-Linear-systems/KJ-4-2-07-b-multians.pg**

This is the third part of a three-part problem.

Consider the system of differential equations

$$\begin{aligned} y_1' &= y_1 + 3y_2, \\ y_2' &= 3y_1 + y_2, \end{aligned}$$

with solutions

$$\begin{aligned} y_1(t) &= c_1e^{4t} + c_2e^{-2t}, \\ y_2(t) &= c_1e^{4t} - c_2e^{-2t}, \end{aligned}$$

for any constants c_1 and c_2.

Rewrite the solution of the equations in vector form as $\vec{y}(t) = c_1\vec{y}_1(t) + c_2\vec{y}_2(t)$.

$$\vec{y}(t) = c_1\begin{bmatrix} \_\_\_ \\ \_\_\_ \end{bmatrix} + c_2\begin{bmatrix} \_\_\_ \\ \_\_\_ \end{bmatrix}$$

14. **(1 pt) Library/FortLewis/DiffEq/3-Linear-systems/03-Linear-systems/KJ-4-2-04-multians.pg**

Suppose

$$\begin{aligned}(t+6)y_1' &= 2ty_1+7y_2, & y_1(1)=0,\\ (t-5)y_2' &= 3y_1+2ty_2, & y_2(1)=2.\end{aligned}$$

(1) This system of linear differential equations can be put in the form $\vec{y}' = P(t)\vec{y}+\vec{g}(t)$. Determine $P(t)$ and $\vec{g}(t)$.

$$P(t) = \begin{bmatrix} \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ \end{bmatrix}$$

$$\vec{g}(t) = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$$

(2) Is the system homogeneous or nonhomogeneous? [?]

(3) Find the largest interval $a < t < b$ such that a unique solution of the initial value problem is guaranteed to exist.

Interval: ________

15. **(1 pt) Library/FortLewis/DiffEq/3-Linear-systems/03-Linear-systems/Lebl-3-3-02a.pg**

(1) Assuming a, b and k are constants, calculate the following derivative. $\frac{d}{dt}\left(\begin{bmatrix} a \\ b \end{bmatrix} e^{kt}\right) = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$

(2) Find a value of k so that $\begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{kt}$ is a solution to $\vec{x}' = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}\vec{x}$.
$k =$ ____

(3) Find a value of k so that $\begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{kt}$ is a solution to $\vec{x}' = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}\vec{x}$.
$k =$ ____

16. **(1 pt) Library/FortLewis/DiffEq/3-Linear-systems/03-Linear-systems/KJ-4-2-07-a-multians.pg**

This is the first part of a three-part problem.

Consider the system of differential equations

$$\begin{aligned}y_1' &= y_1+3y_2,\\ y_2' &= 3y_1+y_2.\end{aligned}$$

Rewrite the equations in vector form as $\vec{y}'(t) = A\vec{y}(t)$.

$$\vec{y}'(t) = \begin{bmatrix} \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ \end{bmatrix}\vec{y}(t)$$

17. **(1 pt) Library/FortLewis/DiffEq/3-Linear-systems/03-Linear-systems/KJ-4-2-19-multians.pg**

Consider the system of higher order differential equations

$$\begin{aligned}y'' &= t^{-1}y' + 5y - tz + (\sin t)z' + e^{2t},\\ z'' &= y - 7z'.\end{aligned}$$

Rewrite the given system of two second order differential equations as a system of four first order linear differential equations of the form $\vec{y}' = P(t)\vec{y}+\vec{g}(t)$. Use the following change of variables

$$\vec{y}(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \\ y_4(t) \end{bmatrix} = \begin{bmatrix} y(t) \\ y'(t) \\ z(t) \\ z'(t) \end{bmatrix}.$$

$$\begin{bmatrix} y_1' \\ y_2' \\ y_3' \\ y_4' \end{bmatrix} = \begin{bmatrix} \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} + \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$$

18. **(1 pt) Library/FortLewis/DiffEq/3-Linear-systems/09-Nonhomogeneous-systems/KJ-4-8-07.pg**

Consider the initial value problem

$$\vec{y}' = \begin{bmatrix} -3 & -2 \\ 4 & 3 \end{bmatrix}\vec{y} + \begin{bmatrix} 4\sin(t) \\ 0 \end{bmatrix}, \qquad \vec{y}(0) = \begin{bmatrix} -4 \\ 0 \end{bmatrix}.$$

(1) Form the complementary solution to the homogeneous equation.

$$\vec{y}_C(t) = c_1\begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix} + c_2\begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$$

(2) Construct a particular solution by assuming the form $\vec{y}_P(t) = (\sin t)\vec{a} + (\cos t)\vec{b}$ and solving for the undetermined constant vectors $\vec{a}$ and $\vec{b}$.

$$\vec{y}_P(t) = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$$

(3) Form the general solution $\vec{y}(t) = \vec{y}_C(t) + \vec{y}_P(t)$ and impose the initial condition to obtain the solution of the initial value problem.

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ \end{bmatrix}$$

19. (1 pt) Library/FortLewis/DiffEq/3-Linear-systems/09-Nonhomogeneous-systems/KJ-4-8-10.pg

As an illustration of the difficulties that may arise in using the method of undetermined coefficients, consider

$$\vec{y}' = \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix} \vec{y} + \begin{bmatrix} e^{6t} \\ 0 \end{bmatrix}.$$

(1) Form the complementary solution to the homogeneous equation.

$$\vec{y}_C(t) = c_1 \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix} + c_2 \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$$

(2) Show that seeking a particular solution of the form $\vec{y}_P(t) = e^{6t}\vec{a}$, where $\vec{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ is a constant vector, does not work. In fact, if $\vec{y}_P$ had this form, we would arrive at the following contradiction:

$a_2 =$ ___ $\cdot a_1$

and

$a_2 =$ ___ $\cdot a_1 -$ ___.

(3) Show that seeking a particular solution of the form $\vec{y}_P(t) = te^{6t}\vec{a}$, where $\vec{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ is a constant vector, does not work either. In fact, if $\vec{y}_P$ had this form, we would arrive at the following contradiction:

$a_2 =$ ___ $\cdot a_1$

and

$a_1 =$ ___

and

$a_2 =$ ___.

20. (1 pt) Library/FortLewis/DiffEq/3-Linear-systems/09-Nonhomogeneous-systems/KJ-4-8-05.pg

Consider the initial value problem

$$\vec{y}' = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \vec{y} + \begin{bmatrix} t \\ 5 \end{bmatrix}, \qquad \vec{y}(0) = \begin{bmatrix} -2 \\ -2 \end{bmatrix}.$$

(1) Form the complementary solution to the homogeneous equation.

$$\vec{y}_C(t) = c_1 \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix} + c_2 \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$$

(2) Construct a particular solution by assuming the form $\vec{y}_P(t) = \vec{a} + \vec{b}t$ and solving for the undetermined constant vectors $\vec{a}$ and $\vec{b}$.

$$\vec{y}_P(t) = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$$

(3) Form the general solution $\vec{y}(t) = \vec{y}_C(t) + \vec{y}_P(t)$ and impose the initial condition to obtain the solution of the initial value problem.

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ \end{bmatrix}$$

21. (1 pt) Library/FortLewis/DiffEq/3-Linear-systems/09-Nonhomogeneous-systems/KJ-4-8-06.pg

Consider the initial value problem

$$\vec{y}' = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \vec{y} + \begin{bmatrix} t \\ e^{2t} \end{bmatrix}, \qquad \vec{y}(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

(1) Form the complementary solution to the homogeneous equation.

$$\vec{y}_C(t) = c_1 \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix} + c_2 \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$$

(2) Construct a particular solution by assuming the form $\vec{y}_P(t) = \vec{a}e^{2t} + \vec{b}t + \vec{c}$ and solving for the undetermined constant vectors $\vec{a}, \vec{b}$, and $\vec{c}$.

$$\vec{y}_P(t) = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$$

(3) Form the general solution $\vec{y}(t) = \vec{y}_C(t) + \vec{y}_P(t)$ and impose the initial condition to obtain the solution of the initial value problem.

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ \end{bmatrix}$$