Hala Al Hajj Shehadeh **MATH336_0001_FA15**
Assignment H.w.5 due 12/01/2015 at 02:31pm EST

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1. **(1 pt) Library/FortLewis/DiffEq/3-Linear-systems/01-Intro-to-systems/KJ-4-1-13-multians.pg**
Let

$$A(t) = \begin{bmatrix} 2 & \ln(|t|) \\ \sqrt{4-t} & e^{3t} \end{bmatrix}.$$

(1) Find $A'(t)$.

$$A'(t) = \begin{bmatrix} \_\_\_\_\_\_ & \_\_\_\_\_\_ \\ \_\_\_\_\_\_ & \_\_\_\_\_\_ \end{bmatrix}$$

(2) Find $A'(t)$.

$$A''(t) = \begin{bmatrix} \_\_\_\_\_\_ & \_\_\_\_\_\_ \\ \_\_\_\_\_\_ & \_\_\_\_\_\_ \end{bmatrix}$$

(3) $A(t)$ is defined for all t in the interval ______

$A'(t)$ is defined for all t in the interval ______

$A''(t)$ is defined for all t in the interval ______

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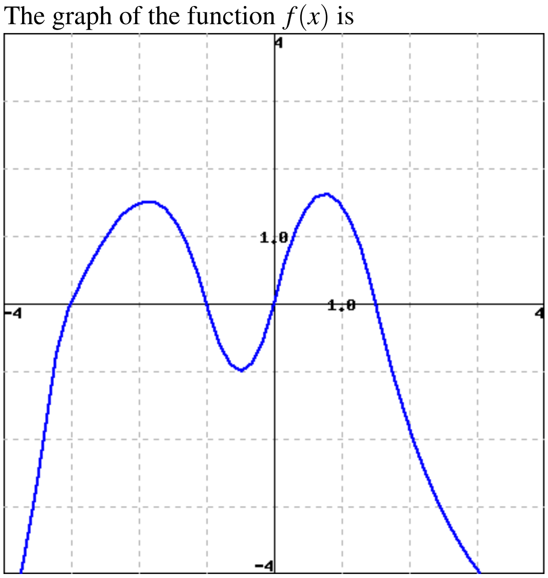
2. **(1 pt) Library/Rochester/setDiffEQ6AutonomousStability-/ur_de_6_1.pg**
The graph of the function $f(x)$ is



(the horizontal axis is x.)
Consider the differential equation $x'(t) = f(x(t))$.

List the constant (or equilibrium) solutions to this differential equation in increasing order and indicate whether or not these equations are stable, semi-stable, or unstable.

____ ?
____ ?
____ ?
____ ?

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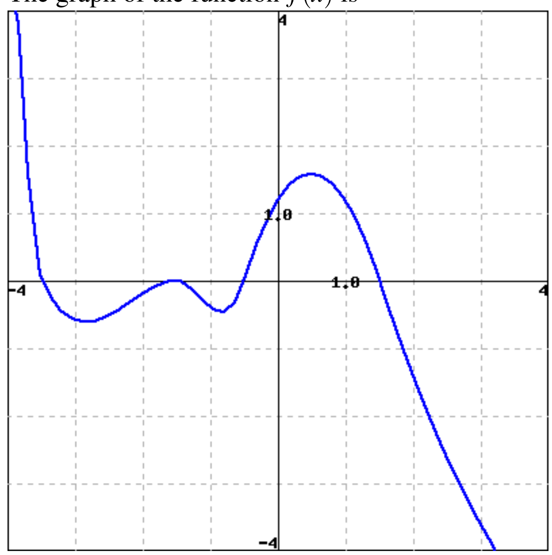
3. **(1 pt) Library/Rochester/setDiffEQ6AutonomousStability-/ur_de_6_2.pg**
The graph of the function $f(x)$ is



(the horizontal axis is x.)
Given the differential equation $x'(t) = f(x(t))$.
List the constant (or equilibrium) solutions to this differential equation in increasing order and indicate whether or not these equations are stable, semi-stable, or unstable.

____ ?
____ ?
____ ?
____ ?

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4. **(1 pt) Library/Rochester/setDiffEQ6AutonomousStability-/ur_de_6_3.pg**
Given the differential equation $x' = -(x+2.5)*(x+0.5)^3(x-1)^2(x-2)$.
List the constant (or equilibrium) solutions to this differential equation in increasing order and indicate whether or not these equations are stable, semi-stable, or unstable. (It helps to **sketch the graph.** xFunctions will plot functions as well as phase planes.)

____ ?
____ ?

____ [?]
____ [?]

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5. **(1 pt) Library/Rochester/setDiffEQ6AutonomousStability-/ur_de_6_4.pg**

Given the differential equation $x'(t) = -x^4 + 3x^3 + 8x^2 - 12x - 16$.

List the constant (or equilibrium) solutions to this differential equation in increasing order and indicate whether or not these equations are stable, semi-stable, or unstable. (It helps to **sketch the graph.** xFunctions will plot functions as well as phase planes.)

____ [?]
____ [?]
____ [?]
____ [?]

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6. **(1 pt) Library/Rochester/setDiffEQ13Systems1stOrder-/ur_de_13_1.pg**

Write the given second order equation as its equivalent system of first order equations.

$$u'' + 8u' + 4u = 0$$

Use v to represent the "velocity function", i.e. $v = u'(t)$.
Use v and u for the two functions, rather than $u(t)$ and $v(t)$. (The latter confuses webwork. Functions like $\sin(t)$ are ok.)

$u' =$ ________________
$v' =$ ________________

Now write the system using matrices:

$$\frac{d}{dt}\begin{bmatrix} \text{u} \\ \text{v} \end{bmatrix} = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}\begin{bmatrix} \text{u} \\ \text{v} \end{bmatrix}.$$

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7. **(1 pt) Library/Rochester/setDiffEQ13Systems1stOrder-/ur_de_13_9.pg**

Consider the following model for the populations of rabbits and wolves (where R is the population of rabbits and W is the population of wolves).

$$\frac{dR}{dt} = 0.05R(1 - 0.00025R) - 0.000703125RW$$

$$\frac{dW}{dt} = -0.05W + 0.000125RW$$

Find all the equilibrium solutions:
(a) In the absence of wolves, the population of rabbits approaches ________.
(b) In the absence of rabbits, the population of wolves approaches ________.
(c) If both wolves and rabbits are present, their populations approach $r =$ ________ and $w =$ ________.

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8. **(1 pt) Library/Rochester/setDiffEQ13Systems1stOrder-/ur_de_13_11.pg**

Solve the system

$$\frac{dx}{dt} = \begin{bmatrix} 8 & -4 \\ 4 & -2 \end{bmatrix} x$$

with the initial value $x(0) = \begin{bmatrix} -11 \\ -10 \end{bmatrix}$.

$$x(t) = \begin{bmatrix} \_\_\_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_\_\_ \end{bmatrix}.$$

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9. **(1 pt) Library/Rochester/setDiffEQ13Systems1stOrder-/ur_de_13_12.pg**

Consider the interaction of two species of animals in a habitat. We are told that the change of the populations $x(t)$ and $y(t)$ can be modeled by the equations

$$\frac{dx}{dt} = 5x - 2y,$$

$$\frac{dy}{dt} = -2x + 2.5y.$$

[?] 1. What kind of interaction do we observe?

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10. **(1 pt) Library/Rochester/setDiffEQ13Systems1stOrder-/ur_de_13_13.pg**

Liam opens a bank account with an initial balance of 500 dollars. Let $b(t)$ be the balance in the account at time t. Thus $b(0) = 500$. The bank is paying interest at a continuous rate of 5% per year. Liam makes deposits into the account at a continuous rate of $s(t)$ dollars per year. Suppose that $s(0) = 500$ and that $s(t)$ is increasing at a continuous rate of 2% per year (Liam can save more as his income goes up over time).

(a) Set up a linear system of the form

$$\frac{db}{dt} = m_{11}b + m_{12}s,$$

$$\frac{ds}{dt} = m_{21}b + m_{22}s.$$

$m_{11} =$ ____,
$m_{12} =$ ____,
$m_{21} =$ ____,
$m_{22} =$ ____.

(b) Find $b(t)$ and $s(t)$.
$b(t) =$ __________,
$s(t) =$ __________.

11. **(1 pt) Library/FortLewis/DiffEq/3-Linear-systems/07-Repeated-eigenvalues/KJ-4-7-29.pg**

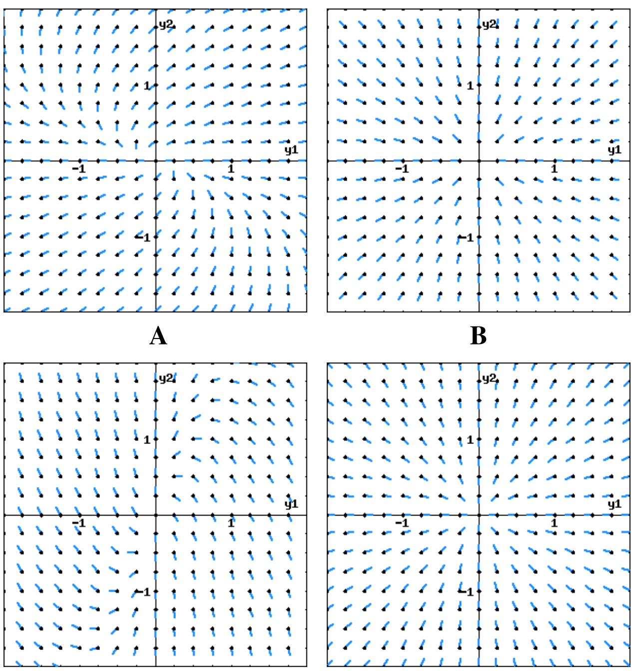
Match each linear system with one of the phase plane direction fields. (The blue lines are the arrow shafts, and the black dots are the arrow tips.)

[?] 1. $\begin{aligned} y_1' &= -y_1 \\ y_2' &= 2y_1 - y_2 \end{aligned}$

[?] 2. $\vec{y}\,' = \frac{1}{2}\begin{bmatrix} -0.5 & 0 \\ 0 & -0.5 \end{bmatrix}\vec{y}$

[?] 3. $\begin{aligned} y_1' &= y_1 + y_2 \\ y_2' &= y_2 \end{aligned}$

[?] 4. $\vec{y}\,' = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\vec{y}$



A B C D

12. **(1 pt) Library/FortLewis/DiffEq/3-Linear-systems/08-Complex-eigenvalues/KJ-4-6-04-multians.pg**

Consider the linear system

$$\vec{y}\,' = \begin{bmatrix} 6 & 4 \\ -10 & -6 \end{bmatrix}\vec{y}.$$

Find the eigenvalues and eigenvectors for the coefficient matrix.

$\lambda_1 =$ ___, $\vec{v}_1 = \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$, and $\lambda_2 =$ ___, $\vec{v}_2 = \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$

13. **(1 pt) Library/FortLewis/DiffEq/3-Linear-systems/08-Complex-eigenvalues/KJ-4-6-13-multians.pg**

Suppose A is a 2×2 real matrix with an eigenvalue $\lambda = 6i$ and corresponding eigenvector

$$\vec{v} = \begin{bmatrix} -1-i \\ 1 \end{bmatrix}.$$

Determine a fundamental set (i.e., linearly independent set) of solutions for $\vec{y}\,' = A\vec{y}$, where the fundamental set consists entirely of *real* solutions.

Enter your solutions below. Use t as the independent variable in your answers.

$\vec{y}_1(t) = \begin{bmatrix} \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \end{bmatrix}$

$\vec{y}_2(t) = \begin{bmatrix} \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \end{bmatrix}$

14. **(1 pt) Library/FortLewis/DiffEq/3-Linear-systems/08-Complex-eigenvalues/KJ-4-6-14-multians.pg**

Suppose A is a 2×2 real matrix with an eigenvalue $\lambda = 1 + 5i$ and corresponding eigenvector

$$\vec{v} = \begin{bmatrix} -1+i \\ i \end{bmatrix}.$$

Determine a fundamental set (i.e., linearly independent set) of solutions for $\vec{y}\,' = A\vec{y}$, where the fundamental set consists entirely of *real* solutions.

Enter your solutions below. Use t as the independent variable in your answers.

$\vec{y}_1(t) = \begin{bmatrix} \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \end{bmatrix}$

$\vec{y}_2(t) = \begin{bmatrix} \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \end{bmatrix}$

15. **(1 pt) Library/FortLewis/DiffEq/3-Linear-systems/08-Complex-eigenvalues/KJ-4-6-20-multians.pg**

Consider the linear system

$$\vec{y}\,' = \begin{bmatrix} 3 & 2 \\ -5 & -3 \end{bmatrix}\vec{y}.$$

(1) Find the eigenvalues and eigenvectors for the coefficient matrix.

$\lambda_1 =$ ___, $\vec{v}_1 = \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$, and $\lambda_2 =$ ___, $\vec{v}_2 = \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$

(2) Find the real-valued solution to the initial value problem

$$\begin{cases} y_1' = 3y_1 + 2y_2, & y_1(0) = 4, \\ y_2' = -5y_1 - 3y_2, & y_2(0) = -10. \end{cases}$$

Use t as the independent variable in your answers.

$y_1(t) =$ ______________________

$y_2(t) =$ ______________________

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16. (1 pt) Library/FortLewis/DiffEq/3-Linear-systems/08-Complex-eigenvalues/KJ-4-6-32.pg

Match each initial value problem with the phase plane plot of its solution. (The arrows on the curves indicate how the solution point moves as t increases.)

[?] 1. $\vec{y}' = \begin{bmatrix} 1 & -0.5 \\ 0.5 & 1 \end{bmatrix} \vec{y}, \quad \vec{y}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$

[?] 2. $\vec{y}' = \begin{bmatrix} -1 & -0.5 \\ 0.5 & -1 \end{bmatrix} \vec{y}, \quad \vec{y}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$

[?] 3. $\vec{y}' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \vec{y}, \quad \vec{y}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$

[?] 4. $\vec{y}' = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} \vec{y}, \quad \vec{y}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$