NYU MathUA-233 Theory of Probability (Summer 2017 Session II)

(Week 1) Mon July 3- Thurs July 6, 2017

1 Week 1 Topics

1.1 Chapter 1: Combinatorial Analysis.

1. The basic and generalized-basic principles of counting (multiplication).

2. Permutations and combinations:

   - There are $n!$ different permutations of n different objects (order matters).
   - There are $\frac{n!}{n_1!n_2!...n_r!}$ different permutations of n objects, for which n_1 are alike, n_2 are alike, ... , and n_r are alike (order matters).
   - There are $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ possible combinations of n objects taken r at a time (order does not matter).
   - There are $\binom{n}{n_1, n_2, \ldots, n_r} = \frac{n!}{n_1!n_2!...n_r!}$ possible divisions of n different objects into r different groups of respective sizes $n_1, n_2, \ldots, n_r$ where $n_1 + n_2 + \cdots + n_r = n$ (order does not matter).
   - $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}, 1 \leq r \leq n$

3. The binomial theorem: $(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$.

4. The multinomial theorem: $(x_1+x_2+\cdots+x_r)^n = \sum_{n_1+\cdots+n_r=n} \binom{n}{n_1, n_2, \ldots, n_r} x_1^{n_1} x_2^{n_2} \ldots x_r^{n_r}$.

1.2 Chapter 2: Axioms of Probability.

1. Set theory: complement; unions; intersections; commutative, associative and distributive laws; DeMorgan's laws (hold for finite, infinite countable, and infinite uncountable sets $\{E_i\}$).

2. Sample space S: Set of *all* possible outcomes of an experiment (the outcome of the experiments under consideration is not predictable with certainty).

3. Event E: A subset of the sample space: so it's a set of possible outcomes of an experiment. If an outcome of an experiment is contained in E, then we say that E has *occurred*.

4. *Limiting relative frequency* definition of probability of an event E ($P(E)$): Repeat the experiment (whose sample space is S) under exactly the same conditions.

$$P(E) = \lim_{n\to\infty} \frac{n(E)}{n},$$

where $n(E)$ is the number of times in the first n repetitions of the experiment that the event E occurs. So $P(E)$ is the limiting relative frequency of E.

5. *Axiomatic* definition of probability: Consider an experiment whose sample space is S. For each event E of S, we assume that a number $P(E)$ (which we call the *probability of the event* E) is defined and satisfies the following axioms:

   - $0 \leq P(E) \leq 1$,
   - $P(S) = 1$,
   - For any sequence of mutually exclusive events $E_1, E_2, \ldots$, $P(\bigcup_{i=1}^{\infty} E_i) = \sum_{i=1}^{\infty} P(E_i)$.

6. Probability as a *measure* on the sample space S.

7. Propositions:

   (a) $P(E^c) = 1 - P(E)$.
   (b) If $E \subset F$, then $P(E) \leq P(F)$.
   (c) $P(E \cup F) = P(E) + P(F) - P(EF)$.
   (d) $P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(EF) - P(EG) - P(FG) + P(EFG)$.
   (e) $P(\bigcup_{i=1}^{n} E_i) = \sum_{i=1}^{n} P(E_i) - \sum_{i_1<i_2} P(E_{i_1}E_{i_2}) + \ldots + (-1)^{r+1} \sum_{i_1<i_2<\cdots<i_r} P(E_{i_1}E_{i_2}\ldots E_{i_r}) + \cdots + (-1)^{n+1} P(E_1E_2\ldots E_n)$, where $\sum_{i_1<i_2<\cdots<i_r} P(E_{i_1}E_{i_2}\ldots E_{i_r})$ is taken over all of the $\binom{n}{r}$ possible subsets of size r of $\{1, 2, \ldots, n\}$.
   (f) Bonferroni's inequality: $P(EF) \geq P(E) + P(F) - 1$.
   (g) The probability that exactly one of the events E or F occurs is $P(E) + P(F) - 2P(EF)$.

8. *Finite* sample spaces with equally likely outcomes: If we assume that all outcomes of an experiment are equally likely (each point in the sample space has the same probability), then for any event E,

$$P(E) = \frac{\text{number of outcomes in } E}{\text{number of outcomes in } S}.$$

2 Reading assignment

- Chapter 1 sections: 1.1, 1.2, 1.3, 1.4, 1.5.
- Chapter 2 sections: 2.1, 2.2, 2.3, 2.4, 2.5.

3 Homework Assignments

1. on Mon July 3 lecture: Page 15 (2, 3, 10, 24, 26). Page 17 (5).
2. on Wed July 5 lecture: Page 48 (1, 2, 3, 6, 8). Page 52 (2, 4, 9).
3. on Thurs July 6 lecture: Page 48 (11, 16, 18, 21, 25, 45, 55). Page 52 (6, 7, 20).

4 Test 1 on Thursday July 6

Name:

1. State the *limiting relative frequency* definition of probability of an event E.

2. State the *axiomatic* definition of probability of an event E.

3. Prove that if $E \subset F$ then $P(E) \leq P(F)$.

4. Suppose that A and B are mutually exclusive events for which $P(A) = 0.3$ and $P(B) = 0.5$. What is the probability that A occurs but B does not?

5. Give a combinatorial explanation of the identity $\binom{n}{r} = \binom{n}{n-r}$.

6. Counting:

   (a) If 4 Americans, 3 French people, and 3 British people are to be seated in a row, how many seating arrangements are possible when people of the same nationality must sit next to each other?

   (b) A student is to answer 7 out of 10 questions in an examination. How many choices has she? How many if she must answer at least 3 of the first 5 questions?

(c) How many 5-digit numbers can be formed from the integers $1, 2, \ldots, 9$ if no digit can appear more than *twice*?

(d) A committee of 6 people is to be chosen from a group consisting of 7 men and 8 women. If the committee must consist of at least 3 women and at least 2 men, how many different committees are possible?